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In Two Parts.

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DECIMAL ARITHMETIC, with the *Extraction* of the SQUARE and CUBE ROOTS.

And also the *Multiplication of Feet and Inches*, commonly called CROSS-MULTIPLICATION.

The Second PART teaching to
Measure all Sorts of SUPERFICIES and SOLIDS, by *Decimals*, by *Cross-Multiplication*, and by *Scale and Compasses*: Also the Works of several Artificers, relating to *Building*; and the Measuring of *Board* and *Timber*. Shewing the common Errors.

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By WILLIAM HAWNEY, *Philomath.*

Recommended by the Rev. Dr. John Harris, *F. R. S.*

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THE PREFACE.

HAVING perused several Books concerning the Mensuration of Superficies and Solids, and the Works of Artificers relating to Building; but not finding any one Book so perfect, as to give any tolerable Satisfaction to a Learner; and I having practised and taught Measuring for several Years, and thereby gain'd Experience and Knowlege in that Art, having learned some Things from one Author, and some Things from another, I began to think of digesting my Thoughts into some such Method as might give a Learner full Satisfaction, without being at the Charge of buying so many Books; and being importun'd thereunto by some Friends, I fell to work, and at last brought them to that Perfection you here find in the following Work.

1. As to the *Decimal Arithmetic*, I have been as brief as the Matter would well bear, to make it plain.

2. As to the Multiplying of Feet and Inches, commonly call'd *Croſs-Multiplication*; my Method differs from that which is uſually taught in other Authors, as being (I think) much ſhorter and plainer.

3. In meaſuring of Superficies and Solids, I have given the Demonſtration of the Rules, which I thought might be very acceptable to the Ingenious; for, indeed, I always look upon the Writing of a Rule without a Demonſtration (in any Part of the Mathematicks) to be but lame and defective; and for want of knowing the Reaſon of the Rule, a Learner may commit great Errors; beſides, when a Learner knows the Reaſon of the Rules, he may retain them better in his Memory. The Rule for meaſuring a Priſmoid and Cylindroid, I had out of Mr. *Everard's Art of Gauging*; but the Reaſon he does not ſhew, neither have I found it in any other Author; but that the Method is true, I have endeavour'd to make plain.

The Demonſtration of the Rules for finding the Area of an Ellipſis and Parabola; alſo the Demonſtration of the Rules for finding the ſolid Content of the Fruſtum of a Cone and Pyramid, the Solidity of a Globe of a Spheroid, a Parabolic Conoid, and of a Parabolic Spindle, and their Fruſtums, I had from the ingenious Mr. *Ward's Young Mathematician's Guide*; where the Curious and ingenious Reader may ſee many other Demonſtrations algebraically perform'd. I have alſo demonſtrated the Rule for finding the Solidity of a Globe out of *Pardic's Elements of Geometry* (Book the 5th, Art. the 33d) publiſh'd in *Engliſh* with many

ny Additions, by the Reverend Dr. *Harris*, *F. R. S.* and the ſame is alſo done out of *Sturmius's Matheſis Enucleata*; ſo that the ingenious Reader may uſe which of thoſe Ways he likes beſt.

The Scale ſuppos'd to be uſed in all the Operations, is the Line of Numbers, commonly call'd, *Gunter's Line*, which is upon the ordinary Two-Feet, or Eighteen-Inch Rules, commonly uſed by the Carpenters, Maſons, *&c.* becauſe I thought it needleſs, as well as impertinent, to write the Uſe of Sliding-rules, or any other particular Scales, they being ſufficiently treated of by ſeveral Authors, *viz.* by the above-nam'd Mr. *Everard*, in his *Art of Gauging* above-mentioned, where you have the Uſe of a Sliding-rule in Arithmetic, Geometry, in Meaſuring of Superficies and Solids, Gauging, *&c.* Likewiſe Mr. *Hunt* has written largely of the Uſes of his Sliding-rule, in Arithmetic, Geometry, Trigonometry, Gauging, Dialing, *&c.* There are ſeveral others who have explain'd the Uſe of their own Rules; ſo that the more curious Readers may find full Satisfaction in thoſe Authors.

One thing I have omitted in the Book, which I think may not be very improperly inſerted in this Place; that is, how to find a Number upon the Line. If the Number you would find conſiſts only of Units, then the Figures upon the Line repreſent the Number ſought: Thus, if the Number be 1, 2, 3, *&c.* then 1, 2, 3, *&c.* upon the Line, repreſents the Number ſought. But if the Number conſiſts of two Figures, that is, of Units and Tens, then the Figure upon the Rule ſtands for the Tens, and

the large Divisions stand for the Units; thus, if 34 were to be found upon the Line, the Figure 3 upon the Line is 30, and 4 of the large Divisions (counted forwards) is the Point representing 34; and if 340 were to be found, it will be at the same Point upon the Line; and if 304 were to be found, then the 3 upon the Line is 300, and four of the smaller Divisions (counted forward) is the Point representing 304. If the Number consists of four Places, or Thousands, then the Figure upon the Line stands for Thousands, and the larger Divisions are Hundreds, the lesser Divisions are Tens, and the tenth Parts of those lesser Divisions are Units. Thus, if 2735 were to be found, then the 2 is 2000; and the 7 larger Divisions (counted forward) is 700 more; and 3 of the lesser Divisions is 30 more; and half of one of the lesser Divisions is 5 more, which is the Point representing 2735. You must remember, that between each Figure upon the Line there are 10 Parts, which I call the larger Divisions; and each of those larger Divisions are subdivided (or supposed so to be) into 10 other Parts, which I call the smaller Divisions; and each of those Parts supposed to be subdivided again into 10 other Parts, *&c.* You must also remember, that if 1, in the Middle of the Line, stands only for 1, then 1 at the upper End will be 10, and 1 at the lower End will only be $\frac{1}{10}$; but if 1 at the lower End signifies 1, then 1 in the Middle stands for 10, and 1 at the upper End is 100, *&c.*

There is one Thing more which I would have my Reader to understand, and that is, How to find all such proportional Numbers made use of in the Proportions about a Circle, and of a Cylinder, and

in other Places; which Thing may be of good Uſe; to know how to correct a Number, which may happen to be falſe printed, or to inlarge any Number to more decimal Places, for more Exactneſs; for tho' I have mention'd what ſuch Numbers are, yet I have not ſhewn how to find them, which a Learner may be a little at a Nonplus to do; tho' they are eaſily ſound by the Rules there laid down. I ſhall therefore give two or three Examples, in this Place, of finding ſuch Numbers, which may enable my Reader to find out the reſt.

And, firſt, let it be requir'd to find the Area of a Circle, whoſe Diameter is an Unit.

By the Proportion of *Van Culen*, if the Diameter be 1, the Circumference will be 3.1415926, *&c.* whereof 3.1416 is ſufficient in moſt Caſes. Then the Rule teaches to multiply half the Circumference by half the Diameter, and the Product is the Area: That is, multiply 1.5708 by .5 (*viz.* half 3.1416 by half 1) and the Product is .7854 which is the Area of the Circle, whoſe Diameter is 1.

Again, If the Area be required when the Circumference is 1, firſt, find what the Diameter will be, thus: As 3.1416 : to 1 : : ſo is 1 to .318309, which is the Diameter when the Circumference is 1. Then multiply half .318309 by half 1; that is .159154 by .5, and the Product is .079577, which is the Area of a Circle whoſe Circumference is 1.

If the Area be given, to find the Side of the Square equal, you need but extract the Square Root of the Area given, and it is done: So the Square Root of .7853 is .88.62, which is the Side of a Square

Square equal when the Diameter is 1. And if you extract the ſquare Root of .079577, it will be .2821, which is the Side of the Square equal to the Circle whoſe Circumference is 1.

If the Side of a Square within a Circle be requir'd, if you ſquare the Semidiameter, and double that Square, and out of that Sum extract the Square Root, that ſhall be the Side of the Square which may be inſcrib'd in that Circle; ſo, if the Diameter of the Circle be 1, then the Half is .5; which, ſquar'd, is .25; and this, doubled, is 5, whoſe Square Root is .7071, the Side of the Square inſcrib'd.

Again, If the Diameter of a Globe be 1, to find the Solidity. In Sect. XI. Chap. II. it is demonſtrated, that the Globe is $\frac{2}{3}$ of a Cylinder of the ſame Diameter and Altitude: Thus if the Cylinder's Diameter be 1, and its Altitude or Length be alſo 1, find the Solidity thereof, and take $\frac{2}{3}$ of it, and that will be the Solidity of the Globe requir'd. Now if the Diameter be 1, the Area of the Circle, or Baſe of the Cylinder, is .7854, (as is above ſhewn) which multiplied by 1, the Altitude of the Cylinder, and the Product is alſo .7854, the Solidity of the Cylinder; $\frac{2}{3}$ whereof is .5236, which is the Solidity of the Globe, whoſe Diameter is 1.

From what has been ſaid, the Reader may eaſily perceive how all other proportional Numbers are found, and may examine them at his Pleaſure.

I ſhall not inlarge any further upon the Matter, but leave the Book to ſpeak for itſelf; and if it prove beneficial to the ingenious Practitioners,

ners, I have my Desire. So, wishing my ingenious Reader good Success in his Endeavours, not doubting but he will reap Profit hereby; which that he may, is the hearty Desire of his Well-wisher,

W. HAWNEY.

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THE

THE
Complete MEASURER.

PART I.

CHAP. I.

Notation of DECIMALS.

A DECIMAL Fraction is an artificial Way of setting down and expressing of Natural or Vulgar Fractions, as whole Numbers: And whereas the Denominators of Vulgar Fractions are divers, the Denominators of Decimal Fractions are always certain: For a Decimal Fraction hath always for its Denominator an Unit, with a Cypher or Cyphers annex'd to it, and must therefore be either 10, 100, 1000, 10000, *&c.* and therefore, in writing down of a Decimal Fraction, there is no Necessity of writing down the Denominator; for by bare Inspection it is certainly known, it consisting of a Unit, with as many Cyphers annexed to it as there are Places (or Figures) in the Numerator.

Example. This Decimal Fraction $\frac{25}{100}$ may be written thus .25, its Denominator being known to be an Unit with two Cyphers; because there are two Figures in the Numerator. In like manner, $\frac{125}{1000}$ may be thus written, .125; and $\frac{3575}{10000}$ thus, .3575; and $\frac{75}{1000}$ thus, .075; and $\frac{65}{10000}$ thus, .0065.

As whole Numbers increase in a decuple or tenfold Proportion, towards the Left Hand, so, on the contrary, Decimals decrease towards the Right Hand in a decuple Proportion, as in the following Scheme.

Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenth Parts.	Hundredth Parts.	Thousandth Parts.	Ten Thousandth Parts.	HundredThousandth Parts.	Millionth Parts, &c.
7	6	5	4	3	2	1	0	1	2	3	4	5	6

Hence it appears, that Cyphers put on the Right Hand of whole Numbers, do increase the Value of those Numbers in a decuple (or tenfold) Proportion; but being annexed to the Right Hand of a Decimal Fraction, do neither increase nor decrease the Value thereof: So $\frac{2500}{10000}$ is equivalent to $\frac{25}{100}$ or .25. And, on the contrary, tho' in whole Numbers, Cyphers prefix'd before them, do neither increase nor diminish the Value; yet Cyphers before a decimal Fraction do diminish its Value in a decuple Proportion: For .25, if you prefix a Cypher before it, becomes $\frac{025}{1000}$ or .025: and .125 is $\frac{00125}{100000}$, by prefixing two Cypher before it, thus, .00125. And therefore, when you are to write a Decimal Fraction, whose Denominator hath more Cyphers than there are Figures in the

the Numerator, they must be supply'd by prefixing so many Cyphers before the Figures of your Numerator; as, suppose $\frac{19}{1000}$ were to be written down, without its Denominator; here, because there are three Cyphers in the Denominator, and but two Figures in the Numerator, therefore prefix a Cypher before 19, and set it down thus, .019.

The Integers are separated from the Decimals several ways, according to Mens Fancies; but the best and most usual Way is by a Point or Period; and if there be no whole Number, then a Point before the Fraction is sufficient: Thus, if you were to write down 317 $\frac{217}{1000}$ it may be thus express'd, 317.217; and 59 $\frac{25}{10000}$ thus, 59.0025; and $\frac{75}{10000}$ thus, .0075, &c.

CHAP. II.

Reduction of DECIMALS.

IN *Reduction of Decimals*, there are three Cases: 1st, To reduce a vulgar Fraction to a Decimal. 2dly, To find the Value of a Decimal in the known Parts of Coin, Weights, Measures, &c. 3dly, To reduce Coin, Weights, Measures, &c. to a Decimal. Of these in their Order.

I. *To reduce a vulgar Fraction to a Decimal.*

The RULE.

As the Denominator of the given Fraction is to its Numerator, so is an Unit (with a competent Number of Cyphers annex'd) to the Decimal required.

Therefore, if to the Numerator given, you annex a competent Number of Cyphers, and divide the Re-

ſult by the Denominator, the Quotient is the Decimal equivalent to the vulgar Fraction given.

Example 1. Let $\frac{3}{4}$ be given, to be reduc'd to a Decimal of two Places, or having 100 for its Denominator.

To 3 (the Numerator given) annex two Cyphers, and it makes 300; which divide by the Denominator 4, and the Quotient is .75, the Decimal requir'd, and is equivalent to $\frac{3}{4}$ given.

Note, That ſo many Cyphers as you annex to the given Numerator, ſo many Places muſt be prick'd off in the Decimal found; and if it ſhall happen, that there are not ſo many Places of Figures in the Quotient, the Deficiency muſt be ſupply'd, by prefixing ſo many Cyphers before the Quotient Figures, as in the next Example.

Example 2. Let $\frac{3}{573}$ be reduced to a Decimal having ſix Places.

To the Numerator annex ſix Cyphers, and divide by the Denominator, and the Quotient is 5235; but it was requir'd to have ſix Places, therefore you muſt prefix two Cyphers before it, and then it will be 005235, which is the Decimal required, and is equivalent to $\frac{3}{573}$.

See the Work of theſe two Examples.

```
4)3.00(.75           573)3.000000(005235
  28                          ....
  ----                    --------
   20                     1350
   20                      2040
  ----                      3210
   ..                     --------
                             345
```

In the ſecond Example there remains 345, which Remainder is very inſignificant, it being leſs than $\frac{1}{100000}$ Part of an Unit, and therefore is rejected.

II. *To*

II. *To find the Value of a Decimal in the known Parts of Money, Weight, Measures,* &c.

The RULE.

Multiply the given Decimal by the Number of Parts in the next inferior Denomination, and from the Product prick off ſo many Places to the Right Hand as there were Places in the Decimal given; and multiply thoſe Figures prick'd off by the Number of Parts in the next inferior Denomination, and prick off ſo many Places as before, and ſo continue to do, till you have brought it to the loweſt Denomination requir'd.

Example 1. Let .7565 of a Pound Sterling be given to be reduc'd to Shillings, Pence, and Farthings.

Multiply by 20, by 12, and by 4, as the Rule directs, and always prick off four Places to the Right Hand, and you will find it make 15 *s.* 1 *d.* 2 *q.* See the Work.

$$\begin{array}{rr}
 & .7565 \\
 & 20 \\
s. & \overline{15.1300} \\
 & 12 \\
d. & \overline{1.5600} \\
 & 4 \\
q. & \overline{2.2400}
\end{array}$$

A more compendious Way of finding the Value of the Decimal of a Pound Sterling.

Double the firſt Figure, (or Place of Primes) and it makes ſo many Shillings; and if the next Figure (or Plaee of Seconds) be 5, or more than 5, for the 5 add another Shilling to the former Shillings; then

for every Unit in the second Place count ten, and to that add the Figure in the third Place, and reckon that so many Farthings; but if they make above 13, abate 1; and if it be above 38, abate 2, and add the remaining Farthings to the Shillings before found.

Example 1. Let .695 of a Pound be reduced to Shillings, Pence, and Farthings.

First, Double your 6, and it makes 12 *s.* then take 5 out of 9, and for that reckon another Shilling, and it makes 13 *s.* and the four remaining is 4 Tens, and the 5 makes 45, which being above 38, you must therefore cast away 2, and there rests 43 Farthings, which is 10 *d.* $\frac{3}{4}$. So the Answer is 13 *s.* 10 *d.* $\frac{3}{4}$.

	l.	*s.*	*d.*
So the Value of	.725 =	14	6
And the Value of	.878 =	17	$6\frac{1}{4}$
And the Value of	.417 =	8	4

And so of any other.

Let .59755 of a Pound Troy be reduced to Ounces, Peny-weights, and Grains.

Multiply by 12, by 20, and by 24, and always prick off five Places towards the Right hand, and you will find the Answer to be 7 *oz.* 3 *pwt.* 13 *gr. ferè.* See the Work.

```
 .59755
     12
-------
7.17060
     20
-------          oz. pwt. gr.
3.41200   Facit  7   3    9.888
     24
-------
 164800
 82400
-------
9.88800
```

Let

Let .43569 of a Ton be reduc'd to Hundreds, Quarters, and Pounds.

Multiply by 20, by 4, and by 28, and the Answer will be 8 *C.* 2*qrs.* 24 lb. *fere.*

```
.43569
    20
----------
8.71380
      4          C.  qrs.  lb.
---------- Facit 8   2     23.9456
2.85520
     28
----------
23.94560
```

Let .9595 of a Foot be reduc'd into Inches and Quarters.

```
.9595
   12
--------
11.5140
      4 Facit 11 Inches, 2 Quarters.
--------
2.0560
```

III. *To reduce the known Parts of Money, Weight, Measure, &c. to a Decimal.*

The RULE.

To the Number of Parts of the lesser Denomination given, annex a competent Number of Cyphers, and divide by the Number of such Parts that are contain'd in the greater Denomination, to which the Decimal is to be brought; and the Quotient is the Decimal sought.

Example

Example 1. Let 6*d*. be reduced to the Decimal of a Pound.

To 6 annex a competent Number of Cyphers, (ſuppoſe 3) and divide the Reſult by 240, (the Pence in a Pound) and the Quotient is the Decimal requir'd.

```
240)6.000(.025
    -----
     1200
    ------ Facit .025
     ....
```

Example 2. Let 3*d*. $\frac{3}{4}$ be reduc'd to the Decimal of a Pound, having ſix Places.

In 3 *d*. $\frac{3}{4}$ there are fifteen Farthings, therefore to 15 annex ſix Cyphers, (becauſe there are to be ſix Places in the Decimal requir'd) and divide by 960, (the Farthings in a Pound) and the Quotient is .015625.

```
96'0)15.00000'0(.015625
          .....
         -------
          540
           600
            240
             480
           -------
             ...
```

Example 3. Let 3 $\frac{1}{4}$ Inches be reduc'd to the Decimal of a Foot, conſiſting of four Places.

In 3 $\frac{1}{4}$ Inches, there are 13 Quarters; therefore to 13 annex four Cyphers, and divide by 48, (the Quarters in a Foot) and the Quotient is .2708.

```
48)13.0000(.2708
   --------
      340
       400
      -----
        16
```

Example

Example 4. Let 9 *C* 1 *qr*. 16 lb. be reduc'd to the Decimal of a Ton, having six Places.

```
C. qu. lb.
 9  1  16        2240)1052.00000|0(469642
 4                     ......
 ——               ——————————————
 37 qrs-                 15600
 28                       21600
 ——                        14400
302 Facit .469642           9600
 75                          6400
———                         ——————
1052 Pounds                 1920
```

CHAP. V.

Addition of DECIMALS.

ADDITION of Decimals is perform'd the same way as Addition of whole Numbers, only you must observe to place your Numbers right, that is, Units under Units, primes under Primes, Seconds under Seconds, &c.

Example. Let 317.25, 17.125, 275.5, 47.3579, and 12.75, be added together into one Sum.

```
    317.25
     17.125
    275.5
     47.3579
     12.75
    ————————
Sum 669.9829
```

This is so plain, that more Examples I think needless.

CHAP.

CHAP. IV.

Subtraction of DECIMALS.

SUBTRACTION of Decimals is perform'd likewise the ſame way as in whole Numbers, reſpect being had to the right placing the Numbers, (as in Addition) as in the following Examples.

(1)

From	212.0137
Subtr.	31.1275
Reſts	180.8862
Proof	212.0137

(2)

From	201.1250
Subtr.	5.5785
Reſts	195.5465
Proof	201.1250

(3)

From	2051.315
Subtr.	79.172
Reſts	1972.143
Proof	2051.315

(4)

From	30.5
Subtr.	7.2597
Reſts	23.2403
Proof	30.5.

Note, If the Number of Places in the Decimals be more in that which is to be ſubtracted, than in that which you ſubtract from, you muſt ſuppoſe Cyphers to make up the Number of Places, as in the fourth Example.

CHAP. V.

Multiplication of DECIMALS.

MULTIPLICATION of Decimals is alſo perform'd the ſame way as Multiplication of whole Numbers; but to know the Value of the Product, obſerve this Rule.

Cut off, or ſeparate by a Comma, or Prick, ſo many Decimal Places in the Product, as there are Places of Decimals in both Factors, *viz.* in the Multiplicand and Multiplier; which I ſhall farther explain in the following Examples.

Let 3.128 be multiply'd by 2.75; multiply the Numbers together, as if they were whole Numbers, and the Product is 8.59375: and becauſe there were three Places of Decimals prick'd off in the Multiplicand, and two Places in the Multiplier, therefore you muſt prick off five Places of Decimals in the Product, as you may ſee by the Work.

```
  3.125
   2.75
 ------
  15625
 21875
 6250
 ------
8.59375
```

Let

Let 79. 25 be multiply'd by .459.

In this Exemple, becaufe two Places of Decimals are prick'd off in the Multiplicand, and three in the Multiplier, therefore there muft be five prick'd off in the Product.

```
   79.25
    .459
 -------
   71325
  39625
 31700
 -------
 36.37575
```

Let .135272 be multiply'd by .00425.

In this Example, becaufe in the Multiplicand are fix Decimal Places, and in the Multiplier five Places; therefore in the Product there muft be eleven Places of Decimals; but when the Multiplication is finifh'd, the Product is but 57490600, *viz.* only eight Places; therefore, in this Cafe, you muft prefix three Cyphers before the Product Figures, to make up the Number of eleven Places: fo the true Product will be .00057490600.

```
      .135272
       .00425
  -----------
       676360
      270544
     541088
  -----------
 .00057490600
```

More

More Examples for Practice.

.001472	.017532
.1045	347
7360	122724
5888	70128
14720	52596
.0001538240	6.083604

279.25	32.0752
.445	.0325
139625	1603760
111700	641504
111700	962256
124.26625	1.04244400

4.443	20.0291
15.98	35.45
35544	1001455
39987	801164
22215	1001455
4443	600873
70.99914	710.031595

7.3564	.75432
.0126	.0356
441384	452592
147128	377160
73564	226296
.09269064	.026853792

Contracted Multiplication of Decimals.

Becauſe in Multiplication of Decimal Parts, and mix'd Numbers, there is no Need to expreſs all the Figures of the Product, but in moſt Caſes two, three, or four Places of Decimals will be ſufficient; therefore, to contract the Work, obſerve this following

RULE.

Write the Unit's Place of the Multiplier under that Place of the Multiplicand, whoſe Place you intend to keep in the Product; then invert the Order of all the other Figures, that is, write them all the contrary Way. Then, in multiplying, always begin at that Figure in the Multiplicand which ſtands over the Figure you are then multiplying withal, and ſet down the firſt Figure of each particular Product directly one under the other: But yet a due Regard muſt be had to the Increaſe ariſing from the Figures on the Right Hand of that Figure in the Multiplicand which you begin to multiply at. This will appear more plain by Examples.

Example 1. Let 2.38645 be multiply'd by 8.2175, and let there be only four Places retained in the Decimals of the Product.

Firſt, according to the Directions, write down the Multiplicand, and under it write the Multiplier, thus; place the 8 (being the Unit's Place of the Multiplier) under 4, the fourth Place of Decimals in the Multiplicand, and write the reſt of the Figures quite contrary to the uſual Way, as in the following Work: Then begin to multiply, firſt the five which is left out, (only with regard to the Increaſe which muſt be carry'd from it, ſaying 8 times 5 is 40, carry 4 in your Mind, and ſay 8 times 4 is 32, and 4 I carry, is 36; and ſet down ſix, and carry 3, and proceed thro' the reſt of the Figures, as in common Multiplication:

Then begin to multiply with 2, ſaying 2 times 4 is 8, for which I carry 1, (becauſe it is above 5) and ſay, 2 times 6 is 12, and 1 that I carry is 13; ſet down 3, and carry 1, and proceed thro' the reſt of the Figures: Then multiply with 1, ſaying, once 6 is 6, for which carry 1, and ſay, once 8 is 8, and 1 is 9; ſet down 9, and proceed: Then multiply with 7, ſaying, 7 times 8 is 56, for which carry 6, (becauſe it is above 55) and ſay, 7 times 3 is 21, and 6 that I carry is 27; ſet down 7, and carry 2, and proceed: Then multiply with 5, ſaying 5 times 3 is 15, for which carry 2, and ſay 5 times 2 is 10, and 2 I carry is 12, which ſet down, and add all the Products together; and the total Product will be 19.6107. See the Work.

```
2.38645
5712.8
-------
19.0916
   4773
    239
    167
     12
-------
19.6107
```

Note, That in multiplying the Figure left out every Time next the Reight Hand in the Multiplicand, if the Product be 5, or upwards to 10, you carry 1; and if it be 15, or upwards to 20, carry 2; and if 25, or upwards to 30, carry 3, *&c.*

I have here ſet down the Work of the laſt Example, wrought by the common Way, by which you may ſee both the Reaſon and Excellency of this Way, all the Figures on the Right Hand the Line being wholly omitted.

2.38645	
8.2175	
11	93225
167	0515
238	645
4772	90
190916	0
19.6106	52875

Example 2. Let 375.13758 be multiply'd by 16.7324, so that the Product may have but four Places of Decimals.

First, set 6, the Unit's Place of the Multiplier, under 5, being the fourth Place of Decimals in the Multiplicand, (becauſe four Places of Decimals were to be prick'd off) and write all the reſt of the Figures backward. Then multiply all the Figures of the Multiplicand by 1, after the common Way. Then begin with the ſecond Figure of the Multiplier 6, ſaying 6 times 8 is 48, for wich I carry 5, (in reſpect of the 8 left out) and 6 times 5 is 30, and 5 that I carry is 35; ſet down 5 and carry 3. and proceed after the common Method. Then begin with 7, the third Figure of the Multiplier, and ſay 7 times 5 is 35, for which carry 4, and ſay 7 times 7 is 49, and 4 I carry is 53; ſet down 3 under the firſt, and carry 5, and proceed as before. Then begin with 3, the fourth Figure of the Multiplier, and ſay 3 times 7 is 21, carry 2, and ſay 3 times 3 is 9, and 2 I carry is 11; ſet down 1 and carry 1, and proceed as before. Then begin with 2, the fifth Figure, and ſay 2 times three is 6, for which I carry 1, and ſay 2 times 1 is 2, and 1 I carry is 3; ſet down 3; and 2 times 5 is 10; ſet down 0, and carry 1, and proceed as before. Then begin with 4 the laſt Figure of the Multiplier, and ſay 4 times 1 is 4, for which I carry nothing, becauſe 'tis leſs than 5; then

then say 4 times 5 is 20; set down 0, and carry 2, and proceed thro' the rest of the Figures of the Multiplicand. Then add all up together and the Product is 6276.9520. See the Work.

375.13758 the Multiplicand
4237.61 the Multiplier revers'd.

37513758 the Product with 1.
22508255 the Product with 6 increas'd with 6 × 8.
2625963 the Product with 7 increas'd with 7 × 5.
112541 the Product with 3 increas'd with 3 × 7.
7503 the Product with 2 increas'd with 2 × 3.
1500 the Product with 4 increas'd with 0.

6276.9520 the Product requir'd.

Let the same Example be repeated, and let only one Place in Decimals be prick'd off.

375.13758 the Multiplicand.
4237.61 the Multiplier inverted.

37514 the Product by 1 with the Increase of 1 × 7.
22508 the Product with 6 increas'd with 6 × 3.
2626 the Product with 7 increas'd with 7 × 1.
113 the Product with 3 increas'd with 3 × 5.
7 the Product with 2 increas'd with 2 × 7.
1 the Increase only of 4 × 3.

6276.9 the Product is the same as before.

More Examples for Practice.

Multiply 395.3756 by .75642; and prick off four Places in Decimals.

395.3756 the Multiplicand.
24657. the Multiplier revers'd.

2767629 the Product by 7 increas'd with 7 × 6.
197688 the Product by 5 increas'd with 5 × 5.
23722 the Product by 6 increas'd with 6 × 7.
1581 the Product by 4 increas'd with 4 × 3.
79 the Product by 2 increas'd with 2 × 5.

299.0699 the Product requir'd.

Let the same Example be repeated, and let there be only one Place of Decimals.

395.3756
24657.

2767 the Product by 7 increas'd with 7 × 3.
198 the Product by 5 increas'd with 5 × 5.
24 the Product by 6 increas'd with 6 × 9 + 6 × 5.
2 the Increase of 4 × 9 + 4 × 3.

299.1 the Product.

Characters, and their Signification.

Note, That this Mark $+$ signifies Addition; as $8+5$, that is 8 more 5, or 8 added to 5; and $8+3+7$ denotes these Numbers are to be added into one Sum.

This Mark $-$ signifies Subtraction, as $9-4$ signifies that 4 is to be taken from 9.

This Mark $\times$ signifies Multiplication, as 7×5 signifies that 7 is to be multiply'd by 5.

This Mark $\div$ signifies Division, as $12 \div 4$ signifies 12 is to be divided by 4.

This Mark $=$ signifies Equality, or Equation; that is, when $=$ is placed between Numbers, or Quantities, it denotes them to be Equal, as $7+5=12$, that is, 7 more 5 is equal to 12; and $15-7=8$, that is, 15 less by 7, is equal to 8, or subtract 7 from 15 and there remains 8.

This Mark : : is the Sign of Proportion, or the Golden Rule, it being always placed betwixt the two middle Terms or Numbers in Proportion, thus, 4 : 20 : : 6 : 30, to be thus read, as 4 is to 20, so is 6 to 30.

CHAP. VI.

Division of Decimals.

DIVISION of Decimals is performed after the same Manner as Division of whole Numbers; but to know the Value or Denomination of the Quotient, is the only Difficulty; for the resolving of which observe either of the following

RULES.

RULES.

I. The firſt Figure in the Quotient muſt be of the ſame Denomination with that Figure in the Dividend which ſtands (or is to be ſuppos'd to ſtand) over the Unit's Place in the Diviſor, at the firſt ſeeking.

II. When the Work of Diviſion is ended, count how many Places of Decimal Parts there are in the Dividend more than in the Diviſor; for that Exceſs is the Number of Places which muſt be ſeparated in the Queſtion for Decimals: But if there be not ſo many Figures in the Quotient, as is the ſaid Exceſs, that Deficiency muſt be ſupply'd with Cyphers in the Quotient, prefix'd before the ſignificant Figures thereof, towards the Left Hand, with a Point before them; ſo ſhall you plainly diſcover the Value of the Quotient.

Theſe following Directions ought alſo to be carefully obſerv'd.

If the Diviſor conſiſts of more Places than the Dividend, there muſt be a competent Number of Cyphers annex'd to the Dividend, to make it conſiſt of as many (at leaſt) or more Places of Decimals than the Diviſor; for the Cyphers added muſt be reckon'd as Decimals.

Conſider whether there be as many decimal Parts in the Dividend as there are in the Diviſor; if there be not, make them ſo many, or more, by annexing of Cyphers.

In dividing of whole or mix'd Numbers, if there be a Remainder, you may bring down more Cyphers, and, by continuing your Diviſion, carry the Quotient to as many Places of Decimals as you pleaſe.

Theſe Things being conſider'd, I ſhall proceed to the Practice of Diviſion of Decimals, which I ſhall endeavour to explain in as familiar and eaſy a Method as poſſible.

Example

Example 1. Let 48 be divided by 144.

In this Example the Diviſor 144 is greater than the Dividend 48; therefore, according to the Directions above, I annex a competent Number of Cyphers, (*viz.* four) with a Point between them, and divide according to the uſual way.

```
144)48.0000(.3333
    ---------
    480
     480
      480
    ---------
       48
```

But, firſt, in ſeeking how often 144 in 48.0 (the firſt three Figures of the Dividend), I find the Unit's Place of the Diviſor to fall under the firſt Place of Decimals; therefore the firſt Figure in the Quotient is in the firſt Place of Decimals: Or, by the ſecond Rule, there being four Places of Decimals in the Dividend, and none in the Diviſor; ſo the Exceſs of decimal Places in the Dividend, above that in the Diviſor, is four; ſo that when the Diviſion is ended, there muſt be four Places of Decimals in the Quotient. See the Work.

Example 2. Let 217.75 be divided by 65.

Firſt, in ſeeking how oft 65 in 217 (the firſt three Figures of the Dividend), I find the Unit's Place of the Diviſor to fall under the Unit's Place of the Dividend; therefore the firſt Figure in the Quotient will be Units, and all the reſt Decimals. Or, by the ſecond Rule, there being two Places of Decimals in the Dividend, and no Decimals in the Diviſor, therefore the Exceſs of decimal Places in the Dividend, above the Diviſor, is two; ſo when the Diviſicn is ended, ſeparate two Places in the Quotient, towards the Right Hand by a Point. See the Work.

65)

65)217.75(3.35

227
325

Example 3. Let 267.15975 be divided by 13.25.

13.25)267.15975(20.163

2159
8347
3975

In this Example, 3, the Unit's Place of the Divisor, falls under 6, the Ten's Place of the Dividend; therefore (by the first Rule) the first Figure in the Quotient is Tens: Or, by the second Rule, the Excess of Decimal Places in the Dividend, above the Divisor, is three; there being five Places of Decimals in the Dividend, and but two in the Divisor, so there must be three Places of Decimals in the Quotient.

Example 4. Let 15.675159 be divided by 375.89.

375,89)15.675159(.0417

63955
263669

546

In

In this Example, 5, in the Unit's Place of the Divisor, falls under 7, the second Place of Decimals in the Dividend; therefore (by the first Rule) the first Figure in the Quotient is in the second Place of Decimals; so that you must put a Cypher before the first Figure in the Quotient; and by the second Rule, the Excess of decimal Places in the Dividend above the Number of decimal Places in the Divisor is 4; for the decimal Places in the Dividend is 6, and the Number of Places in the Divisor but two; therefore there must be four Places of Decimals in the Quotient. But the Division being finish'd after the common Way, the Figures in the Quotient are but three; therefore you must prefix a Cypher before the significant Figures.

Example 5. Let 72.1564 be divided by .1347.

```
.1347)72.1564(535.68
         . . .
      ———————
          4806
           7654
            9190
             11080
             ——————
               304
```

In this Example, the Divisor being a Decimal, the first Figure thereof falls under the Ten's Place in the Dividend; therefore the Units (if there had been any) should fall under the Hundred's Place in the Dividend, and so the first Figure in the Quotient is Hundreds. And, by the second Rule, there being four Places of Decimals in the Dividend, and as many in the Divisor, so the Excess is nothing; but in dividing I put two Cyphers to the Remainders, and continue the Division to two Places further; so I have two Places of Decimals. See the Work.

Example 6. Let .125 be divided by .0457.

```
.0457).1250000(2.735
          ....
       --------
       914
       ---
       3360
       3199
       ----
        1610
        1371
        ----
         3390
         2285
         ----
          105
```

In this Example, the Unit's Place of the Divisor (if there had been any) would fall under the Unit's Place of the Dividend; therefore the first Figure of the Quotient is Units. And, by the second Rule, there being seven Places of Decimals in the Dividend, and but four Places in the Divisor, so the Excess is three; therefore there must be three Places of Decimals in the Quotient.

I shall set down only the Work of some few Examples more, and so proceed to *Contracted Division*.

```
.00456).0000059791(.00131.
                 .  ..
        --------------
                 1419
                  511
                  ---
                   55
```

Let

Let it be divided by 282.

282)1.0000000(.0035461 *ferè.*

1540
1300
1720
280

.325).400000(1.2307

750
1000
2500

225

.042)495.00000(11785.71

75
330
360
240
300
60

18

Division of DECIMALS *contracted.*

IN Division of Decimals the common Way, when the Divisor hath many Figures, and it is requir'd to continue the Division till the Value of the Remainder be but small, the Operation will sometimes be large and tedious, but may be excellently contracted by the following Method.

The RULE.

By the first Rule of this Chapter, (*page* 20.) find what is the Value of the first Figure in the Quotient; then, by knowing the first Figure's Denomination, you may have as many or as few Places of Decimals as you please, by taking as many of the Left Hand Figures of the Divisor as you think convenient for the first Divisor; and then take as many Figures of the Divident as will answer them; and in dividing, omit one Figure of the Divisor at each following Operation. A few Examples will make it plain.

Example 1. Let 721.17562 be divided by 2.257432; and let there be three Places of Decimals in the Quotient.

```
2.25743)721.175|62(319.467
  .....  677229
         ------
          43946
          22574
         ------
          21372
          20317
         ------
           1055
            903
         ------
            152
            135
         ------
             17
             15
         ------
              2
```

In this Example, the Unit's Place of the Divisor falls under the Hundred's Place in the Dividend, and it is requir'd. that three Places of Decimals be in the Quotient, so there must be six Places in all, that is, three Places of whole Numbers, and three Places of Decimals. Then, because I can have the Divisor in the first six Figures of the Dividend, I cut off the 62 with a Dash of the Pen, as useless; then I seek how oft the Divisor in the Dividend, and the Answer is three times; put 3 in the Quotient, and multiply and subtract as in common Division, and the Remainder is 43946. Then prick off the 3 in the Divisor, and seek how often the remaining Figures may be had in 43946, the Remainder, which can be but once; put 1 in the Quotient, and multiply and subtract, and the next Remainder is 21372. Then prick off the 4 in the Divisor, and seek how often the remaining Figures may be had in 21372, which will be 9 times; put 9 in the Quotient; multiply thus, saying 9

 times

times 4 is 36, for which I carry 4, (in respect of the 4 last prick'd off) and 9 times 7 is 63, and 4 is 67; set down 7, and carry 6, and so proceed till the Division be finish'd, always respecting the Increase made from the Figures prick'd off. Observe the Work which will better inform you than many Words.

2.25743)721.17562(319.467

677229	
43946	6
22574	3
21372	32
20316	87
1055	450
902	972
152	4780
135	4458
17	03220
15	80201
1	23019

I have set down the Work of this last Example at large, according to the common Way, that thereby the Learner may see the Reason of the Rule, all the Figures on the Right Side the perpendicular Line being wholly omitted.

Example

Example 2. Let 5171.59165 be divided by 8.758615; and let it be requir'd, that four Places of Decimals be prick'd off in the Quotient.

```
8.758615)5171.5916|5(590.4577
 ......
         43793075
         --------
          7922841
          7882754
          -------
            40087
            35034
            -----
             5053
             4379
             ----
              674
              613
              ---
               61
               61
               --
               ..
```

In this Example, I can't have 8, the first Figure in the Divisor, in 5, the first Figure of the Dividend; so that the Unit's Place of the Divisor falls under the Hundred's Place of the Dividend; so that there will be seven Figures in the Quotient, that is, three of whole Numbers, and four of Decimals; therefore there must be 7 Figures in the Divisor, (because the Number of Places in the Divisor and Quotient will be equal) and there must be eight Places in the Dividend; so that I cut off the Figure 5 with a Dash, as useless. Thus having proportion'd the Dividend to the Divisor, and both to the Number of Places or Figures desir'd in the Quotient, I proceed to divide as before, saying how often 8 in 51, which will be 5 times; put 5 in the Quotient, and multiply and subtract,

and the Remainder is 7922841. Then I prick off the first Figure in the Divisor, 5, and seek how often the remaining Figures of the Divisor in the aforesaid Remainder, which I find nine times; put 9 in the Quotient, and multiply thereby, saying 9 times 5 (the Figure prickd off) is 45, for which I carry 5, and say 9 times 1 is 9, and 5 I carry is 14; set down 4, and carry 1, and proceed to multiply the rest of the Figures, and subtract, and the Remainder will be 40087. Then prick off the Figure 1, and seek how often 87586 in the Remainder 40087, the Answer will be 0; so put 0 in the Quotient, and prick off the Figure 6, and seek how often 8758 in 40087, which will be four times; put 4 in the Quotient, and multiply, saying 4 times 6 (the Figure last prick'd off) is 24, for which I carry 2, and say 4 times 8 is 32, and 2 I carry is 34; set down 4, and carry 3; multiply the rest of the Figures, and subtract as before, and so proceed after the same manner, until all the Figures of the Divisor be prick'd off, to the last Figure. See the Work.

Example 3. Let 25.1367 be divided by 217.3543, and let there be five Places of Decimals in the Quotient.

In this Example, 7, the Unit's Place of the Divisor, falls under 1, the first Place of Decimals; therefore the first Figure of the Quotient is in the first Place of Decimals; so the Quotient will be all Decimals. Then, because the Quotient Figures, and the Figures of the Divisor will be of an equal Number, dash off the 43 in the Divisor, and the 7 in the Dividend, as useless, and divide as before.

217.35|43

```
217.35|43)25.136|7(.11564
          21735
          -----
           3401
           2174
          -----
           1227
           1087
          -----
            140
            130
          -----
             10
              8
          -----
              2
```

Altho' I have hitherto given Directions for proportioning the Divisor and Dividend, so as to bring into the Quotient what Number of Decimals you please, yet there is no absolute Necessity for it; but you may carry on your Division to what Degree you please, before you begin to prick off the Figures of the Divisor, in order to contract the Work, as in the following Examples, where it is not requir'd to prick off any determinate Number of Decimals, but it may be done according to Discretion.

```
2.756756)7414.76717(2689.67118
         5513512
         -------
         19012551
         16540536
         --------
         24720157
         22054048
         --------
          2666109
          2481080
          -------
           185029
           165405
           ------
            19624
            19297
            -----
              327
              276
              ---
               51
               28
               --
               23
               22
               --
                1
```

12.34254)514.75498(41.705757
... 4937016

2105338
1234254

871084
863978

7106
6171

935
864

71
62

9
8

1

CHAP. VII.

Extraction of the SQUARE ROOT.

If a Square Number be given;

TO find the Root thereof, that is, to find out ſuch a Number, as being multiply'd into itſelf, the Product ſhall be equal to the Number given; ſuch Operation is call'd, *The Extraction of the Square Root*; which to do, obſerve the following Directions.

1ſt,

1st, You must point your given Number; that is, make a Point or Prick over the Unit's Place, another upon the Hundred's, and so upon every second Figure rhroughout.

2dly, Then seek the greatest square Number in the first Point towards the Left Hand, placing the square Number under the first Point, and the Root thereof in the Quotient, and subtract the said square Number from the first Point, and to the Remainder bring down the next Point, and call that the Resolvend.

3dly, Then double the Quotient, and place it, for a Divisor, on the Left Hand of the Resolvend; and seek how often the Divisor is contained in the Resolvend, (reserving always the Unit's Place) and put the Answer in the Quotient, and also on the Right Hand Side of the Divisor; then multiply by the Figure last put in the Quotient, and subtract the Product from the Resolvend, (as in common Division) and bring down the next Point to the Remainder, (if there be any more) and proceed as before.

A TABLE *of Squares and Cubes, and their Roots.*

Root	1	2	3	4	5	6	7	8	9
Squ.	1	4	9	16	25	36	49	64	81
Cub.	1	8	27	64	125	216	343	521	729

Example 1. Let 4489 be a Number given, and let the square Root thereof be requir'd.

4489.67
36

127.889 Resolvend.
889 Product.

. . .

First,

First, Point the given Number, as before directed; then, by the little Table aforegoing, seek the greatest square Nnmber in 44, (the first Point to the Left Hand) which you will find to be 36, and 6 the Root; put 36 under 44, and 6 in the Quotient, and subtract 36 from 44, and there remains 8. Then to that 8 bring down the other Point 89, placing it on the Right Hand, so it makes 889 for a Resolvend; then double the Quotient 6, and it makes 12, which place on the Left Hand for a Divisor, and seek how often 12 in 88, (reserving the Unit's Place) the Answer is 7 times; which put in the Quotient, and also on the Right Hand Side of the Divisor, and multiply 127 by 7, as in common Division, and the Product is 889, which subtracted from the Resolvend, there remains nothing; so is your Work finish'd; and the square Root of 4489 is 67; which Root if you multiply by itself, that is 67 by 67, the Product will be 4489, equal to the given square Number, and prove the Work to be right.

Example 2. Let 106929 be a Numbtr given, and let the square Root thereof be requir'd.

```
  .  .  .
  106929(.327
   9
  ------
62)169 Resolvend.
   124 Product.
  ------
647)4529 Resolvend.
    4529 Product.
    ------
    ....
```

First, Point your given Number, as before directed, putting a Point upon the Units, Hundreds, and Tens of Thousands; then seek what is the greatest square Number in 10, (the first Point) which by the little

Table you will find to be 9, and 3 the Root thereof; put 9 under 10, and 3 in the Quotient; then ſubtract 9 out of 10, and there remains 1; to which bring down 69, the next Point, and it makes 169 for the Reſolvend; then double the Quotient 3, and it makes 6, which place on the Left Hand of the Reſolvend for a Diviſor, and ſeek how often 6 in 16; the Anſwer is twice; put two in the Quotient, and alſo on the Right Hand of the Diviſor, making it 62. Then multiply 62 by the 2 you put in the Quotient, and the Product is 124; which ſubtract from the Reſolvend, and there remains 45; to which bring down 29, the next Point, and it makes 4529 for a new Reſolvend. Then double the Quotient 32, and it makes 64, which place on the Left Side of the Reſolvend for the Diviſor, and ſeek how oft 64 in 452, which you'll find 7 times; put ſeven in the Quotient, and alſo on the Right Hand of the Diviſor, making it 647, which multiply'd by the 7 in the Quotient, it makes 4529, which ſubtracted from the Reſolvend, there remains nothing. So 327 is the Square Root of the given Number.

Example 3. Let 2268741 be a ſquare Number given, the Root whereof is requir'd.

```
               .  .  .  .
               2268741)1506.23
              1
             -----
            25)126
               125
             -----
         3006)18741
              18036
             -----
        30122)70500
              60244
             -----
      301243)1025600
             .903729
             -------
Remains      .121871
```

Having

Having pointed the given Number as before directed, ſeek what is the greateſt ſquare Number in the firſt Point 2, which is one; put 1, the Square, under 2, and 1, the Root thereof, in the Quotient; ſubtract 1 from 2, and there remains 1; to which bring down the next Point, 26, and ſet on the Right Hand, making it 126; double the 1 in the Quotient; which makes 2; ſet 2 on the Left Hand for a Diviſor, and aſk how often 2 in 12, which will be 5 times; put 5 in the Quotient, and alſo on the Right Hand of the Diviſor, making it 25; multiply (as in common Diviſion) 25 by 5, and ſubtract the Product 125 from 126, and there remains 1. Bring down the next Point, 87, and it makes 187 for a new Reſolvend; and double the 15 in the Quotient, it makes 30 for a new Diviſor. Then ſeek how often 30 in 18, which you can't have; ſo that you muſt put 0 in the Quotient, and alſo on the Right Hand of the Diviſor, and bring down the next Point, and it makes 18741 for another new Reſolvend. Then ſeek how often 300 in 1874, which will be 6 times; put 6 in the Quotient, and alſo on the Right Hand of the Diviſor; multiply and ſubtract, and the Remainder will be 705. Now, if you have a Mind to find the Value of the Remainder, you may annex Cyphers, by two at a time, to the Remainders, and ſo proſecute the Work to what Number of decimal Parts you pleaſe; thus, to 705 annex two Cyphers, and it will make 70500, and the Quotient doubled, is 3012 for a Diviſor: Then ſeek how often 3012 in 7050, (rejecting the Unit's Place) which will be twice; put 2 in the Quotient, and alſo on the Right Hand of the Diviſor, and multiply and ſubtract before, and the Remainder will be as 10256; to which annex two Cyphers, and proceed as before, and you will get a 3 in the Quotient next. So the ſquare Root of the given Number is 1506.23, which being ſquared, or multiply'd by itſelf, and the laſt Remainder added, will make the given Number as follows.

```
          1506.23
          1506.23
         ---------
           451869
          301246
         903738
       7531150
      150623
      -----------
      2268728.8129
The Remainder add—12.1871
      -----------
Proof 2268741.0000
```

Some more Examples for Practice.

```
                 . . . .
*Example* 1.     7596796(2756.228 Root.
                 4
                 ---
               47)359
                  329
                  ----
              545)3067
                  2725
                  -----
             5506)34296
                  33036
                  ------
            55122)126000
                  110244
                  -------
           551242)1575600
                  1102484
                  --------
          5512448)47311600
                  44099584
                  --------
                   3212016
```

Example

Example 2.
```
       · · · · · ·
       751417.5745(866.4 Root.
       64
       --
   166)1114
        996
       -----
  1726)11817
       10356
       ------
 17328)146157
       138624
       -------
173364)753345
       693456
       -------
         59889
```

If the given Number be a mix'd Number, *viz.* consisting of a whole Number and a Decimal together, make the Number of decimal Places even, that is, 2, 4, 6, 8, &c. that so there may a Point fall upon the Unit's Place of the whole Numbers, as in this last Example, and in that following.

Example 3. Let 656714.37512 be given, to find the ſquare Root.

$$
\begin{array}{rl}
 & \dot{6}5\dot{6}7\dot{1}4.\dot{3}7\dot{5}1\dot{2}0(810.379 \text{ Root.} \\
 & 64 \\ \hline
161) & 167 \\
 & 161 \\ \hline
16203) & 61437 \\
 & 48609 \\ \hline
162067) & 1282851 \\
 & 1134469 \\ \hline
1620749) & 14838220 \\
 & 14586741 \\ \hline
\text{Remains} & 251479
\end{array}
$$

In this Example there are five Places of Decimals; therefore put a Cypher to it, to make it even, that ſo there may a Point fall upon 4, the Unit's Place.

To find the Square Root of a Fraction.

If it be a decimal Fraction, the Work differs nothing from the Examples aforegoing, only you muſt be mindful to point your given Number aright; for (as was before directed) the Number of Places muſt always be made even, and then begin to point at the Right Hand, as in whole Numbers.

If it be a vulgar Fraction, it muſt be reduc'd to a Decimal, by the firſt Rule of the ſecond Chapter.

I ſhall give an Example or two in each Caſe, and ſo conclude this Chapter.

Let

Let .125 be a decimal Fraction given, whoſe ſquare Root is requir'd; and let it be requir'd to have four Places of Decimals in the Root.

```
 . . . .
.12500000(.3535
 9
 ———
65)350
   325
 ————
703)2500
    2109
 —————
7065)39100
     35325
 ——————
      3775
```

In this Example there muſt be five Cyphers annex'd, becauſe two Places in the Square make but one in the Root.

Let the ſquare Root of .00715 be requir'd.

```
 . . .
.0071 50).084
  64
 ————
164)750
    656
 ————
     94
```

In this a Cypher is added to make the Place even.

Let $\frac{7}{8}$ be a vulgar Fraction given, whose square Root is requir'd.

8)7000	(.87500000(.9354
64	81
60	183)650
56	549
40	1865)10100
40	9325
..	18704)77500
	74816
	2684

Reduce this $\frac{7}{8}$ to a Decimal, it makes .875; to which annex Cyphers, and extract the square Root, as if it was a whole Number. So the Root is .9354.

Let $\frac{1}{960}$ be a vulgar Fraction, whose square Root is requir'd.

96|0)3.000000	(.00312500(.0559 Root.
	25
288	
	105)625
120	525
96	
	1109)10000
240	9981
192	
	19
480	
480	
...	

In

In extracting the Root of this, because the first Point consists of Cyphers, there must be a Cypher put first in the Quotient.

To prove this Rule, square the Root, and to the Product add the Remainder, as was before directed. To square a Number, is to multiply it by itself; and to cube it, is to multiply the Square of the Number by the Number itself.

CHAP. VIII.

Extraction of the CUBE ROOT.

TO extract the Cube Root, is nothing else but to find such a Number, as being first multiply'd into itself, and then into that Product, produceth the given Number; which to perform, observe these following Directions.

1st, You must point your given Number, beginning with the Unit's Place, and make a Point, or Dot, over every third Figure towards the Left Hand.

2dly, Seek the greatest Cube Number in the first Point, towards the Left Hand, putting the Root thereof in the Quotient, and the said Cube Number under the first Point, and subtract it therefrom, and to the Remainder bring down the next Point, and call that the Resolvend.

3dly, Triple the Quotient, and place it under the Resolvend; the Unit's Place of this under the Ten's Place of the Resolvend; and call this the triple Quotient.

4thly, Square the Quotient, and triple the Square, and place it under the triple Quotient; the Units of this

this under the Ten's Place of the triple Quotient, and call this the triple Square.

5*thly*, Add theſe two together, in the ſame Order as they ſtand, and the Sum ſhall be the Diviſor.

6*thly*, Seek how often the Diviſor is contain'd in the Reſolvend, rejecting the Unit's Place of the Reſolvend, (as in the Square Root) and put the Anſwer in the Quotient.

7*thly*, Cube the Figure laſt put in the Quotient, and put the Unit's Place thereof under the Unit's Place of the Reſolvend.

8*thly*, Multiply the Square of the Figure laſt put in the Quotient, into the triple Quotient, and place the Product under the laſt, one Place more to the Left Hand.

9*thly*, Multiply the triple Square by the Figure laſt put in the Quotient, and place it under the laſt, one Place more to the Left Hand.

10*thly*, Add the three laſt Numbers together, in the ſame Order as they ſtand, and call that the Subtrahend.

Laſtly, Subtract the Subtahend from the Reſolvend; and if there be another Point, bring it down in the Remainder, and call that a new Reſolvend, and proceed in all reſpects as before.

Example

Example 1. Let 314432 be a Cubick Number, whose Root is required.

```
 .  .
314432(68 Root
216
———
 98432 Resolvend.
———
    18 Triple Quotient of 6.
   108 Triple Square of the Quotient 6.
———
  1098 Divisor.
———
     512 Cube of 8, the last Figure of the Root.
    1152 The Square of 8, by the triple Quotient.
   864   The triple Square of the Quotient 6 by 8.
———
  98432 The Subtrahend.
———
  .....
```

After you have pointed the given Number, seek what is the greatest Cube Number in 314, the first Point, which, by the former little Table. (*Page* 34.) you will find to be 216, which is the nearest that is less than 314, and its Root is 6; which put in the Quotient, and 216 under 314, and subtract it therefrom, and there remains 98; to which bring down the next Point, 432, and annex to 98; so will it make 98432 for the Resolvend. Then triple the Quotient 6, it makes 18, which write down the Unit's Place, 8, under 3, the Ten's Place of the Resolvend. Then square the Quotient 6, and triple that Square, and it makes 108, which write under the triple Quotient, one Place toward the Left Hand; then add those two Numbers together, and they make 1098 for the Divisor. Then seek how often the Divisor is contain'd in the Resolvend, (rejecting the Uuit's Place thereof) that is, how often 1098 in 9843, which is 8 times;

8 times; put 8 in the Quotient, and the Cube thereof below the Divisor, the Unit's Place under the Unit's Place of the Resolvend. Then square the 8 last put in the Quotient, and multiply 64, the Square thereof, by the triple Quotient, 18; the Product is 1152; set this under the Cube of 8, the Units of this under the Tens of that. Then multiply the triple Square of the Quotient by 8, the Figure last put in the Quotient, the Product is 864; set this down under the last Product, a Place more to the Left Hand. Then draw a Line under those three, and add them together, and the Sum is 98432, which is called the Subtrahend; which being subtracted from the Resolvend, the Remainder is nothing; which shews the Number to be a true cubick Number, whose Root is 68; that is, if 68 be cub'd, it will make 314432.

For, if 68 be multiplied by 68, the Product will be 4624; and this Product, multiply'd again by 68, the last Product is 314432, which shews the Work to be right.

	68
	68
	544
The Work	408
	4624
	68
	36992
	27744
The Proof.	314432

Example 2. Let the Cube Root of 5735339 be requir'd.

After you have pointed the given Number, seek what is the greatest Cube Number in 5, the first Point, which

which (by the little Table, *page* 34.) you will find to be 1; which place under 5, and 1, the Root thereof, in the Quotient; and subtract 1 from 5, and there remains 4; to which bring down the next Point, it makes 4735 for the Resolvend. Then triple the 1, and it makes 3; and the Square of 1 is 1, and the Triple thereof is 3; which set one under another, in their Order, and added, makes 33 for the Divisor. Seek how often the Divisor in the Resolvend, and proceed as in the last Example.

5735339(179 Root.
1

4735 Resolvend.

3 The Triple of the Quotient 1, the first Figure.
3 The triple Square of the Quotient 1.

33 The Divisor.

343 The Cube of 7, the second Figure of the Root.
147 The Square of 7, multipl. in the triple Quot. 3.
21 The triple Squ. of the Quot. multiply'd by 7.

3913 The Subtrahend.

822339 The new Resolvend.

51 The Triple of the Quot. 17, the two first Fig.
867 The triple Square of the Quotient 17.

8721 Divisor.

729 The Cube of 9, the last Figure of the Root.
4131 The Square of 9, multip. by the trip. Quot. 51.
7803 The triple Square of the Quotient 867 by 9.

822339 The Subtrahend

In this Example, 33, the first Divisor seems to be contain'd more than seven times in 4735, the Resolvend; but if you work with 9, or 8, you will find, that the Subtrahend will be greater than the Resolvend.

Some more Examples for Practice.

32461759(319 The Root.
27

5461 Resolvend.

9 The triple of 3.
27 The triple Square of 3.

279 the Divisor.

1 The Cube of 1, the second Figure.
9 The triple Quotient, by the Square of 1.
27 The triple Square, multipl. by 1, the 2d Figure.

2791 The Subtrahend.

2670759 A new Resolvend.

93 The Triple of 31.
2883 The triple Square of 31.

28923 The Divisor.

729 The Cube of 9, the last Figure.
7533 The Square of 9, by 93 the triple Quotient.
25947 The triple Square 2883 by 9.

2670759 The Subtrahend.

....

8640

84604519(439	The Root
64	
20604	Resolvend,
12	The Triple of 4.
48	The triple Square of 4.
492	The Divisor.
27	The Cube of 3.
108	The Square of 3, by the triple Quotient.
144	The triple Square by 3.
15507	The Subtrahend.
5097519	The Resolvend.
129	The Triple of 43.
5547	The triple square of 43.
55599	The Divisor.
729	The Cube of 9.
10449	The Square of 9 by 129.
49923	The triple Square by 9.
5097519	The Subtrahend.
....	

259697989(638	
216	
43697	Resolvend.
18	The Triple of 6.
108	The Triple Square of 6.
1098	The Divisor.
27	The Cube of 3, the 2d Figure.
162	The triple Square of 3 by 18.
324	The triple Square 108 by 3.
34047	The Subtrahend.
9650989	Resolvend.
189	The Triple of 63.
11907	The triple Square of 63.
119259	The Divisor.
512	The Cube of 8.
12096	The Square of 8 by 189.
95256	The triple Square 11907 by 8.
9647072	The Subtrahend.
3917	The Remainder.

259 7056(295.9
8

17917 The Resolvend.
6 The Triple of 2.
12 The triple Square of 2.

126 The Divisor.

729 The Cube of 9, the 2d Figure.
486 The Square of 9 by 6.
108 The triple Square by 9.

16389 The Subtrahend.

1528056 The Resolvend.
87 The Triple of 29.
2523 The triple Square of 29.

25317 The Divisor.

125 The Cube of 5, the 3d Fig.
2175 The Square of 5 by 87.
12615 The triple Square by 5.

1283375 The Subtrahend.

244681000 The Resolvend.

885 The Triple of 295.
261075 The tripleSquare of 295.

2611635 The Divisor.
729 The Cube of 9, the last Figure.
71685 The Square of 9 by 885.
2349675 The triple Square by 9.
235685079 The Subtrahend.
8995921 The Remainder.

In this Example I annex 3 Cyphers to the Remainder, which makes the 3d Resolvend; by which means I bring one Place of Decimals. And so you may proceed to more decimal Places at Pleasure, by annexing three Cyphers to the next Remainder, and carrying on the Work as before.

```
 .   .   .   .
220698|10125(2805
 8
-----
14069  Resolvend.
-----
    6  The Triple of 2.
   12    The triple Square of 2.
-----
  126    The Divisor.
-----
  512    The Cube of 8.
  384    The Square of 8 by 6.
  96     The triple Square by 8.
-----
13952  The Subtrahend.
      117810125  New Resolvend.
      ---------
        84   The Triple of 28.
      2352   The triple Square of 28.
      -----
     23604   Divisor.
     -----
       840 The Triple of 280.
    235200 The triple Square of 280.
    -------
   2352840 New Divisor.
   -------
       125  Cube of 5.
     21000  Square of 5 by 840.
   1176000  Triple Square by 5.
   -------
 117810125  Subtrahend.
 ---------
 .........
```

In this Example 13952 being subtracted from the Resolv. 14069, the Remainder is 117; to which bring down 810, the third Point, and it makes 117810, for a new Resolv. and the next Divisor is 23604, which you cannot have in the said Resolv. (the Unit's Place being rejected); so you must put 0 in the Quotient, and seek a new Divisor (after you have brought down your last Point to the Resolvend); which new Divisor is 2352840; which you will find to be contained 5 times. So proceed to finish the rest of the Work.

93759.

```
93759·575070(45.42
64
29759 The Resolvend.
   12 The Triple of 4, the first Figure.
   48 The triple Square of 4.
  492 The Divisor.
     125 The Cube of 5, the 2d Figure.
    300 The Square of 5 by 12, the triple Quotient.
   240 The triple Square by 5.
  27125 The Subtrahend.
  2634575 The Resolvend.
     135 The Triple of 45.
    6075 The triple Square of 45.
    60885 The Divisor.
       64 The Cube of 4.
     2160 The Square of 4 by 135.
   24300 The triple Square by 4.
  2451664 The Subtrahend.
   182911070 The Resolvend.
        1362 The Triple of 45.4.
     618348 The triple Square of 45.4.
     6184842 The Divisor.
            8 The Cube of 2.
         5448 The Square of 2 by 1362.
      236696 The triple Square of 2 by 2.
   123724088 The Subtrahend.
    59186982 The Remainder.
```

In extracting the Cube Root of a mix'd Number, always obſerve to make the decimal Part to conſiſt of either three, ſix, nine, &c. Places, that is, always to conſiſt of even Points, as in the laſt Example, where the decimal Places were five; to which I annex'd a Cypher to make up ſix, and ſo I proceed to point it; and by that means I have a Point falls upon the Unit's Place of whole Numbers, which you muſt always obſerve.

To extract the Cube Root *out of a Fraction.*

This is the ſame to do as in whole Numbers, obſerve but the foregoing Directions for the true pointing thereof; for, as was before directed, the Decimal muſt always conſiſt of three, ſix, nine, &c. Places; and if it be not ſo, it muſt be made ſo, by annexing of Cyphers, as is above ſaid.

If the Cube Root of a vulgar Fraction be requir'd, you muſt firſt reduce it to a Decimal, and then extract the Root thereof.

Examples of each follow.

Example

Example 1. Let the Cube Root of .401719179 be required.

.401719179(.737	Root.
343	
58719	Resolvend.
21	Triple of 7.
147	Triple Square of 7.
1491	Divisor.
27	Cube of 3.
189	Square of 3 by 21.
441	Triple Square by 3.
46017	Subtrahend.
12702179	Resolvend.
219	Triple of 73.
15987	Triple Square of 73.
160089	Divisor.
343	Cube of 7.
10731	Square of 7 by 219.
111909	Triple Square by 7.
11298553	Subtrahend.
1406236	Remainder.

Example

Example 2. Let the Cube Root of .0001416 be requir'd.

.0001416oo(.052 Root.
125

16600 Resolvend.

15 The Triple of 5.
75 Triple Square of 5.

765 Divisor.

8 Cube of 2.
60 The Square of 2 by 15.
150 Triple Square by 2.

15608 Subtrahend.

992 Remainder.

Example 3. Let $\frac{5}{276}$ be a vulgar Fraction, whose Cube Root is requir'd.

By the first Rule of Chapter II. reduce the vulgar Fraction to a Decimal,

276)5.000000000(.018115942

2240
320
440
1640
2600
1160
560

8

.01811

.018115942(.262 Root.
8

10115 Resolvend.

6 Triple of 2.
12 Triple Square of 2.

126 Divisor.

216 Cube of 6.
216 Square of 6 by the Triple of 2.
72 The triple Square by 6.

9576 Subtrahend.

539942 Resolvend.

78 Triple of 26.
2028 Triple Square of 26.

20358 Divisor.

8 Cube of 2.
312 Square of 2 by 78.
4056 Triple Square 2028 by 2.

408728 Subtrahend.

131214 Remainder.

You may prove the Truth of the Work, by cubing the Root found, as was shew'd in the first Example; and if any thing remains, add it to the said Cube, and the Sum will be the given Number, if the Work is rightly performed.

I will

I will shew the Proof of the fifth Example, (*pag.* 50.) the given Number being 259697989, whose Root is 638, it being a surd Number, there remains 3917.

	638
	638
	5104
	1914
	3828
The Square	407044
	638
	3256352
	1221132
	2442264
The Cube	259694072
The Remainder add	3917
Proof equal to the given Number	259697989

CHAP. IX.

Multiplication of Feet, Inches, and Parts.

IN the multiplying of Feet, Inches, &c. I shall endeavour to lay down such easy and familiar Rules, as may easily be understood by the meanest Capacity.

Example 1. Let 7 Feet 9 Inches be multiply'd by 3 Feet 6 Inches.

F.	I.	
7	9	
3	6	
23	3	*Pts.*
3	10	6
27	1	6

First, Multiply 9 Inches by 3, saying, 3 times 9 is 27 Inches, which make 2 Feet 3 Inches; set down 3 under Inches, and carry 2 to the Feet, saying, 3 times 7 is 21, and 2 that I carry make 23; set down 23 under the Feet.

Then begin with 6 Inches, saying, 6 times 9 is 54 Parts, which is 4 Inches and 6 Parts; set down 6 Parts, and carry 4, saying, 6 times 7 is 42, and 4 that I carry is 46 Inches, which is 3 Feet 10 Inches; which set down, and add all up together, and the Product is 27 Feet 1 Inch 6 Parts.

Example 2. Let 75 Feet 7 Inches be multiply'd by 9 Feet 8 Inches.

F.	I.	
75	7	
9	8	
680	3	
50	4	8
730	7	8

First, Multiply by 9 Feet, saying, 9 times 7 is 63, which is 5 Feet 3 Inches; set down 3, and carry 5, saying, 9 times 5 is 45, and 5 I carry is 50; set down 0, and carry 5, saying, 9 times 7 is 63, and 5 is

I will shew the Proof of the fifth Example, (*pag.* 50.) the given Number being 259697989, whose Root is 638, it being a surd Number, there remains 3917.

	638
	638
	5104
	1914
	3828
The Square	407044
	638
	3256352
	1221132
	2442264
The Cube	259694072
The Remainder add	3917
Proof equal to the given Number	259697989

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First, Multiply 9 Inches by 3, saying, 3 times 9 is 27 Inches, which make 2 Feet 3 Inches; set down 3 under Inches, and carry 2 to the Feet, saying, 3 times 7 is 21, and 2 that I carry make 23; set down 23 under the Feet.

Then begin with 6 Inches, saying, 6 times 9 is 54 Parts, which is 4 Inches and 6 Parts; set down 6 Parts, and carry 4, saying, 6 times 7 is 42, and 4 that I carry is 46 Inches, which is 3 Feet 10 Inches; which set down, and add all up together, and the Product is 27 Feet 1 Inch 6 Parts.

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F.	I.	
75	7	
9	8	
680	3	
50	4	8
730	7	8

First, Multiply by 9 Feet, saying, 9 times 7 is 63, which is 5 Feet 3 Inches; set down 3, and carry 5, saying, 9 times 5 is 45, and 5 I carry is 50; set down 0, and carry 5, saying, 9 times 7 is 63, and 5 is

is 68; ſet down 68, and proceed to multiply by 8 Inches, ſaying, 8 times 7 is 56; the Twelves in 56 are four times; and 8 remains: ſet 8 a Place to the Right-hand, and carry 4: Then multiply 75 by 8, and the Product is 600, and 4 that I carry is 604, which divided by 12, the Quotient is 50 Feet, and 4 remains; ſet down 50 Feet 4 Inches, and add all up together, and you will find the Product 730 Feet 7 Inches 8 Parts.

I will repeat the laſt Example again, and ſhew another Way to work it, which, I think, is better, and more expeditious, when there are more Figures than one in the Feet; thus,

F.	I.	
75	7	
9	8	
680	3	
25	2	4
25	2	4
730	7	8

Multiply by 9 Feet, firſt, as above directed; then, inſtead of multiplying by 8 Inches, let the 8 Inches be parted into ſuch aliquot or even Parts of a Foot, as you find to be contain'd in that Figure; if you take ſuch Parts of the Multiplicand, and add them to the former Product, the Sum will give the Anſwer: Thus, 8 Inches may be parted into 4, and 4, becauſe 4 is the third Part of 12. So, if you take the third Part of 75 Feet 7 Inches, and ſet it down twice, and add all together, the Sum will be 730 Feet 7 Inches 8 Parts, the ſame as before; thus, ſay how often 3 in 7, which is twice; ſet down 2; then, becauſe twice 3 is 6, ſay, 6 out of 7, and there remains 1, for which you muſt add 10 to the 5, and it makes 15; then the Threes in 15 are 5 times; ſet down 5; and, becauſe 3 times 5 is 15, there is 0 remains. Then go

to the 7 Inches, ſaying, the Threes in 7 are twice; ſet down 2 in the Inches; and becauſe twice 3 is but 6, take 6 out of 7, and there remains 1 Inch, which is 12 Parts; then the Threes in 12 are 4 times, and 0 remains. So the third Part of 75 Feet 7 Inches, is 25 Feet 2 Inches 4 Parts; which ſet down again, and add all together, the Sum is 730 Feet 7 Inches 8 Parts; the ſame as before.

Example 3. Let 97 Feet 8 Inches be multiply'd by 8 Feet 9 Inches.

F.	I.
97	8
8	9
781	4
48	10
24	5
854	7

Begin, firſt, to multiply by 8 Feet, ſaying, 8 times 8 is 64 Inches, that is, 5 Feet 4 Inches; ſet down 4 Inches, and carry 5, ſaying, 8 times 7 is 56, and 5 I carry is 61; ſet down 1, and carry 6, ſaying, 8 times 9 is 72, and 6 I carry is 78, which ſet down: Then, inſtead of multiplying by 9 Inches, take the aliquot Parts of 12 which 9 makes, which is 6 and 3; 6 Inches being half 12, and 3 the fourth Part; therefore take the half of 97 Feet, 8 Inches, which is 48 Feet 10 Inches; and becauſe 3 is half 6, you may take the half of 48 Feet 10 Inches, which is 24 Feet 5 Inches; add all up together, and the Sum is 854 Feet 7 Inches. See the Work, as above.

Example 4. Let 75 Feet 9 Inches be multiply'd by 17 Feet 7 Inches.

F.	I.	
75	9	
17	7	
525		
75		
25	3	P.
18	11	3
8	6	
4	3	
1331	11	3

In this Example, becauſe there are more than 12 Feet in the Multiplier, therefore I firſt multiply the 75 by 17 Feet; then becauſe the aliquot Parts in 7 Inches are 4 and 3, that is, a third and a fourth, I take the third Part of 75 Feet 9 Inches, which is 25 Feet 3 Inches, and the fourth Part thereof is 18 Feet 11 Inches 3 Parts; then the aliquot Parts of 9 Inches are 6 and 3, that is, half and a fourth; therefore I take half 17 Feet, which is 8 Feet 6 Inches, and the fourth Part is 4 Feet 3 Inches (not meddling with the 7 Inches, becauſe that was multiply'd into the 9 before); then add all theſe together, and the Sum is 1331 Feet 11 Inches 3 Parts,

Example

Example 5. Let 87 Feet 5 Inches be multiply'd by 35 Feet 8 Inches.

F.	I.	P.
87	5	
35	8	
435		
261		
29	1	8
29	1	8
11	8	0
2	11	0
3117	10	4

Work here as in the laſt Example. After you have multiply'd the Feet, then take the aliquot Parts of 8 Inches, which is two Thirds; therefore take the third Part of 87 Feet 5 Inches and ſet it down twice. Thus the 3d Part of 87 Feet 5 Inches is 29 Feet 1 Inch 8 Parts; ſet this down twice: Then the aliquot Parts of 5 Inches are 4 and 1, that is, a 3d Part and a 12th Part; therefore take a 3d Part of 35, which is 11 Feet 8 Inches, and a 12th Part of 35 is 2 Feet 11 Inches; ſet all theſe one under another, and add them together, and the Sum is 3117 Feet 10 Inches 4 Parts.

Example 6. Let 259 Feet 2 Inches be multiply'd by 48 Feet 11 Inches,

F.	I.	P.
259	2	
49	11	
2072		
1036		
129	7	
86	4	8
21	7	2
8	0	0
12677	6	10

First, multiply the Feet; then take the aliquot Parts of 11, which will be 6, 4, and 1, that is, an half, a third, and a twelfth; therefore take the half of 259 Feet 2 Inches, which is 129 Feet 7 Inches, and a third Part is 86 Feet 4 Inches 8 Parts, and the twelfth Part of 259 Feet 2 Inches is 21 Feet 7 Inches 2 Parts; or (because 1 is the fourth Part of 4.) you may more readily take the fourth Part of 86 Feet 4 Inches 8 Parts, which is also 21 Feet 7 Inches 2 Parts; then 2 Inches are the sixth of 12, take the sixth of 48 Inches will be 8 Inches, which place under the Inches; then add all together, and the Sum is 12677 Feet 6 Inches 10 Parts. See the foregoing Work.

I shall set down only the Working of some few Examples in Feet and Inches, and then proceed to multiply Feet, Inches, and Parts, &c.

	F.	I.	
	179	3	
	38	10	
	1432		
	537		P.
	89	7	6
	59	9	0
	9	6	0
Product	6960	10	6

	F.	I.	
	246	7	
	46	4	
	1476		
	984		P.
	82	2	4
	15	4	0
	11	6	0
Product	11425	0	4

	F.	I.	
	246	7	
	36	9	
	1476		
	738		P.
	123	3	6
	61	7	9
	12	0	0
	9	0	0
Product	9061	11	3

	F.	I.	
	257	9	
	39	11	
	2313		
	771		P.
	128	10	6
	85	11	0
	21	5	9
	19	6	0
	9	9	0
Product	10288	6	3

Example

Example 11. Let 7 Feet 5 Inches 9 Parts be multiply'd by 3 Feet 5 Inches 3 Parts.

F.	I.	P.		
7	5	9		
3	5	3		
22	5	3	S	
3	1	4	9	T.
	1	10	5	3
25	8	6	2	3

In this Example, I firſt begin with 3 Feet, and thereby multiply 7 Feet 5 Inches 9 Parts: Firſt, I ſay, 3 times 9 is 27 Parts, that is, 2 Inches and 3 Parts; ſet down 3 under the Parts, and carry 2, ſaying, 3 times 5 is 15, and 2 I carry is 17, that is, 1 Foot 5 Inches; ſet down 5 Inches, and carry 1, and ſay, 3 times 7 is 21, and 1 I carry is 22; ſet down 22 Feet. Then begin with 5 Inches, ſaying, 5 times 9 is 45, which is 45 Seconds, which make 3 Parts and 9 Seconds; ſet down 9 Seconds a Place towards the Right-hand, and carry 3 Parts, ſaying, 5 times 5 is 25, and 3 I carry is 28, which is 2 Inches and 4 Parts; ſet down 4 Parts, and carry 2, ſaying, 5 times 7 is 35, and 2 I carry is 37, which is 3 Feet 1 Inch; ſet down 3 Feet 1 Inch, and begin to multiply by 3 Parts, ſaying, 3 times 9 is 27 Thirds, that is, 2 Seconds and 3 Thirds; ſet down 3 Thirds, and carry 2, ſaying, 3 times 5 is 15, and 2 I carry is 17, that is, 1 Part and 5 Seconds; ſet down 5 Seconds, and carry 1, ſaying, 3 times 7 is 21, and 1 I carry is 22, which is 1 Ineh and 10 Parts, which ſet down, and add all up, and the Product is 25 Feet 8 Inches 6 Parts 2 Seconds 3 Thirds.

Note, That in multiplying Feet, Inches, and Parts, *&c.* if Feet be multiply'd by Feet, the Product is Feet; and Feet multiply'd by Inchess, the Product is Inches;

and the twelfth Part is Feet; and Parts multiply'd by Feet, the Product is Parts, and the twelfth Part thereof is Inches; Parts multiply'd by Inches, the Product is Seconds, and the twelfth Part thereof is Parts; and Parts multiply'd by Parts, the Product is Thirds, and the twelfth Part thereof is Seconds. So that if you begin to multiply Parts by Feet in the firſt Row, and Parts by Inches in the ſecond Row, and Parts by Parts in the third Row, the firſt Figure in every Row will ſtand a Place more towards the Right-hand, as you may ſee in the laſt Example.

Example 12. Let 37 Feet 7 Inches 5 Parts be multiply'd by 4 Feet 8 Inches 6 Parts.

F.	I.	P.		
37	7	5		
4	8	6		
150	5	8	S	
12	6	5	8	
12	6	5	8	T
1	6	9	8	6
177	1	5	0	6

Firſt, I multiply by 4 Feet, ſaying, 4 times 5 is 20, which is 1 Inch 8 Parts; ſet down 8, and carry 1, ſaying, 4 times 7 is 28, and 1 I carry is 29, which is 2 Feet 5 Inches; ſet down 5 Inches, and carry 2, ſaying, 4 times 7 is 28, and 2 I carry is 30; ſet down 0, and carry 3, and ſay, 4 times 3 is 12, and 3 is 15; ſet down 15. Then I begin with 8 Inches; but, becauſe the Feet in the Multiplicand are more than 12, it will be the beſt Way to work for the aliquot Parts of 8; ſo here I work for 4 Inches, and ſet that down twice, 4 being the third Part of 12; therefore take the third Part of 37 Feet 7 Inches 5 Parts, which is twelve Feet ſix Inches five Parts eight Seconds; ſet this down twice. Then begin with 6 Parts; but, inſtead of multiplying, take half 37 Feet 7 Inches 5 Parts,

5 Parts, (because 6 is half 12) and set it a Place more to the Right-hand: Thus, the half of 37 Feet is 18, which I must count 18 Inches, because the Multiplier is 6 Parts; so the half of 37 Feet 7 Inches 5 Parts, is 1 Foot 6 Inches 9 Parts 8 Seconds 6 Thirds; which set down, and add all up together, and the Sum is 177 Feet 1 Inch 5 Parts 0 Seconds 6 Thirds.

Example 13. Let 311 Feet 4 Inches 7 Parts be multiply'd by 36 Feet 7 Inches 5 Parts.

F.	I.	P.		
311	4	7		
36	7	5		
1866				
933			S	
103	9	6	4	
77	10	1	9	T
8	7	9	6	4
2	1	11	4	7
12	0	0	0	0
1	0	0	0	0
	9	0	0	0
11402	2	4	11	11

In this Example, because the Feet both in the Multiplier and Multiplicand are compound Numbers, I first multiply the Feet one by the other; then take the aliquot Parts of 7 Inches, which are 4 Inches and 3, that is, a third and a fourth Part; so take the teird Part of 311 Feet 4 Inches 7 Parts, which is 103 F. 9 I. 6. P. 4 S. and the fourth Part is 77 F. 10 I. 1 P. 9 Sec set these down one under another, the Feet under the other Feet; then the aliquot Parts of 5 Parts are 4 and 1, that is, a third and twelfth Part; so the third Part of 311 Feet 4 Inches 7 Parts is 103 F. 9 I. 6 P. 4 Sec. but because the Multiplier is Parts, it must be set a Place to the Right-hand, that

that is, the 103 muſt be Inches, which is 8 Feet 7 Inches; therefore I ſet down 8 F. 7 I 9 P. 6 Sec. 4 Th. Then, becauſe 1 Inch is a fourth Part or 4 Inches, therefore I take a fourth Part of 8 F. 7 I. 9 P. 6 Sec. 4 Th. which is 2 F. 1 I. 11 P. 4 Sec. 7 Th. which is the ſame as if I had taken a twelfth Part of 311 F. 4 I. 7 P. Then for 4 Inchei in the Multiplicand, inſtead of Multiplying 36 Feet by it, take a third Part, becauſe 4 Inches is the third Part of 12; ſo the third Part of 36 is 12 Feet, and the aliquot Parts of 7 Parts are 4 and 3, that is, a third and a fourth; ſo the third Part of 36 is 12, which now is 12 Inches, that is, 1 Foot, and the fourth Part is 9 Inches; add all theſe together, and the Sum will be 11402 Feet 2 Inches 4 Parts 11 Seconds 11 Thirdc.

Example 14. Let 8 Feet 4 Inches 3 Parts 5 Seconds 6 Thirds be multiply'd by 3 Feet 3 Inches 7 Parts 8 Seconds 2 Thirds.

	F.	I.	P.	S.	T.				
	8	4	3	5	6				
	3	3	7	8	2				
	25	0	10	4	6				
	2	1	0	10	4	6			
			10	6	0	2	6		
			5	6	10	3	8	0	
				1	4	8	6	11	0
Prod.	27	7	3	5	1	8	8	11	0

In this laſt Example there is no Difficulty, if you do but obſerve the former Directions, and ſet every Row a Place more to the Right Hand.

I

I ſhall only ſet down the Working of ſome few Examples more, and ſo conclude this Chapter.

F.	I.	P.		
321	7	3		
9	3	6		
2894	5	3	S	
80	4	9	9	T
13	4	9	7	6
2988	2	10	4	6

F.	I.	P.		
42	7	8		
7	3	6		
298	5	8		
10	7	11	S	
1	9	3	10	
310	10	10	10	

F.	I.	P.		
124	7	9		
14	6	2		
496				
124			S	
62	3	10	6	T
1	8	9	3	6
7	0	0	0	0
1	2	0	0	0
	7	0	0	0
	2	6	0	0
1809	1	1	9	6

F.	I.	P.		
259	10	8		
18	5	4		
2072				
259			S	
86	7	6	8	
21	7	10	8	T
7	2	7	6	8
9	0	0	0	0
6	0	0	0	0
1	0	0	0	0
4793	6	0	10	8

F.	I.	P.		
267	7	10		
25	9	7		
1335				
534				
133	9	11	S	
66	10	11	6	
11	1	9	11	T
1	10	3	7	10
12	6	0	0	0
2	1	0	0	0
1	0	6	0	0
	8	1	0	0
6905	0	7	0	10

F.	I.	P.		
317	9	7		
37	5	9		
2219				
951			S	
105	11	2	4	
26	5	9	7	T
13	2	10	9	6
6	7	5	4	9
18	6	0	0	0
9	3	0	0	0
1	6	6	0	0
	3	1	0	0
11910	9	11	1	3

THE

THE Complete Measurer, &c.

PART II.

CHAP. I.

The Mensuration of SUPERFICIES.

Superficial Figures are all such as have only Length and Breadth, not having any commensurable Thickness.

§ I. *Of a* SQUARE.

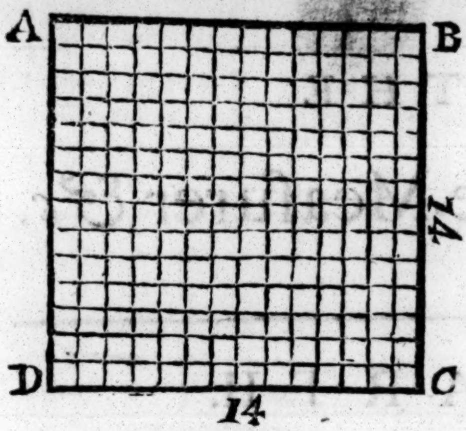
A SQUARE is a Geometrical Figure, having four equal Sides, and as many right (or square) Angels. To find the superficial Content thereof, this is

The RULE.

Multiply the Side into itself, and the Product is the Content.

Let

Let ABCD be a Geometrical Square given, each Side being 14 Feet, Yards, Poles, or other Meaſure; multiply 14 by itſelf, and the Product is 196, which is the ſuperficial Content.



14
14
———
56
14
———
196 Product.

By Scale and Compaſſes.

Extend the Compaſſes from 1, in the Line of Numbers, to 14; the ſame Extent will reach from the ſame Point, turn'd forward to 196.

Demonſtration. Let each Side of the given Square be divided into 14 equal Parts, and Lines drawn from one another, croſſing each other within the Square; ſo ſhall the whole great Square be divided into 196 little Squares, as you may ſee in the Figure above, equal to the Number of Square Feet, Yards, or Poles, or other Meaſures, by which the Side was meaſur'd.

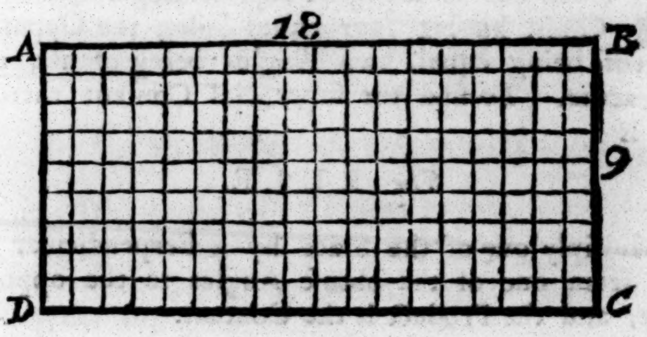
§ II. *Of*

§ II. *Of a* PARALLELOGRAM, *or Long Square.*

A Parallelogram is a Figure having four Sides, and as many Right Angles, the oppoſite Sides thereof being equal and parallel. To find the ſuperficial Content thereof, this is

The RULE.

Multiply the Length by the Breadth, and the Product is the ſuperficial Content.



Length	18
Breadth	9
Product	162

Let ABCD be a long Square, the Length thereof 18 Feet, and the Breadth 9 Feet; which multiply'd together, the Product is 162, the ſuperficial Content thereof.

By Scale and Compaſſes.

Extend the Compaſſes in the Line of Numbers from 1 to 9, the ſame Extent will reach from 18 down to 162, the ſquare Feet.

 Demon-

Demonſtration. If the Sides A B and CD be each divided into 18 equal Parts, repreſenting 18 Feet; and the Lines A D and B C each divided into 9 equal Parts, and Lines drawn from Point to Point, croſſing each other within the Figure; thoſe Lines will make thereby ſo many little Squares as there are ſquare Feet, *viz.* 162.

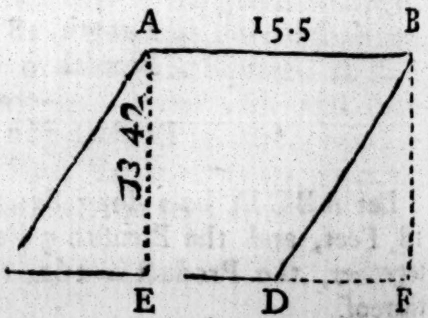
§ III. *Of a* Rhombus.

A Rhombus is a Figure repreſenting a Quarry of Glaſs, having four equal Sides, the Oppoſites thereof being equal, two Angles being obtuſe, and two acute. To find the ſuperficial Content thereof, this is

The RULE.

Multiply one of the Sides by a Perpendicular let fall from one of the obtuſe Angles to the oppoſite Side, and the Product is the Content.

Perpend.	13.42
The Side	15.5
	6710
	6710
	1342
Product	208.010



Let A B C D be a Rhombus given, whoſe Sides are each 15.5 Feet, and the Perpendicular E A is 13.42, which multiply'd together, the Product is 208.010; which is the ſuperficial Content of the Rhombus, that is, 208 Feet and one hundredth Part of a Foot.

By Scale and Compasses.

Extend the Compasses from 1 to 13.42. that Extent will reach from 15.5, the same Way to 208 Feet the Content.

Demonstration. Let CD be extended out to F, makeing DF equal to CE, and draw the Line BF; so shall the Triangle DBF be equal to the Triangle ACE: For DF and CE are equal, and BF is equal to AE, because AB and CF are parallel. Therefore the Parallelogram ABEF is equal to the Rhombus ABCD.

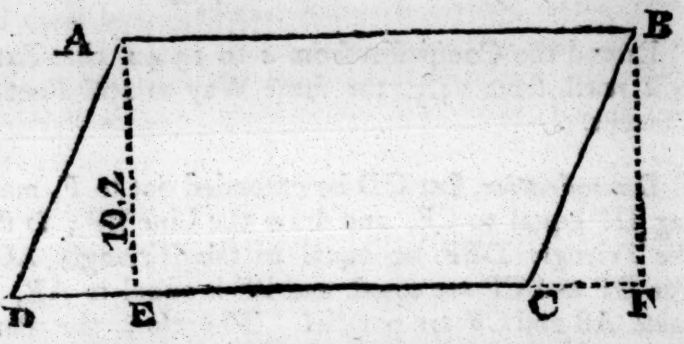
§ IV. *Of a* Rhomboides.

A Rhomboides is a Figure having four Sides, the opposite whereof are equal and parallel; and also four Angles, the opposite whereof are equal. To find the superficial Content thereof, this is

The RULE.

Multiply one of the longest Sides thereof by the Perpendicular let fall from one of the obtuse Angles to one of the longest Sides, and the Product is the Content.

$$\begin{array}{r} 19.5 \\ 10.2 \\ \hline 390 \\ 195 \\ \hline 198.90 \end{array}$$



Let ABCD be a Rhomboides given, whoſe longeſt Sides, AB or CD, is 19.5 Feet, and the Perpendicular AE is 10.2; which multiply'd together, the Product is 198.9, that is, 198 ſuperficial Feet and 9 tenth Parts, the Content.

Demonſtration. If DC be extended to F, making CF equal to DE, and a Line drawn from B to F; ſo will the Triangle CBF be equal to the Triangle ADE, and the Parallelogram AEFB be equal to the Rhomboides ABCD; which was to be proved.

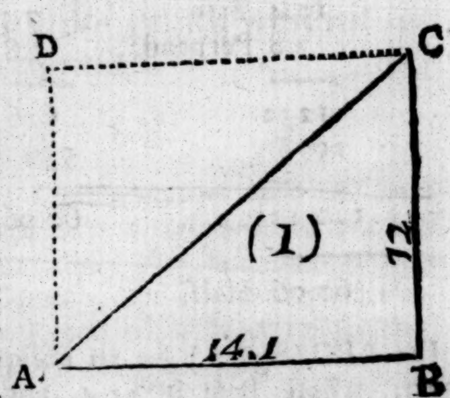
§ V. *Of a* TRIANGLE.

A Triangle is a Figure having three Sides and three Angles. Triangles are either right-angled or oblique-angled. Right-angled Triangles are ſuch as have one Right Angle. Oblique-angled Triangles are ſuch as have their Angles either acute or obtuſe. An obtuſe Angle is greater than a Right Angle, that is, it is more than 90 Degrees; and an acute Angle is leſs than a Right Angle. To find the ſuperficial Content thereof, this is

The

The RULE.

Let the Triangle be of what kind ſoever, multiply the Baſe by half the Perpendicular, or half the Baſe by the whole Perpendicular; or, multiply the whole Baſe by the whole Perpendicular, and take half the Product; any of theſe three Ways will give the Content.

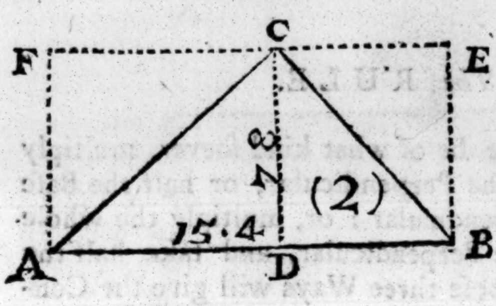


Let ABC be a Right-angled Triangle, whoſe Baſe is 14.1 Feet, and the Perpendicular 12 F. Multiply 14.1 by 6, half the Perpendicular, and the Product is 84.6 Feet, the Content. Or, multiply 14.1 by 12, the Product is 169.2; the half thereof is 84.6, the ſame as before.

14.1 Baſe.	14.1 Baſe.
6 Half Perpendicular.	12 Perpendicular.
84.6 Product.	169.2 Product.
	84.6 Half.

By Scale and Compaſſes.

Extend the Compaſſes from 2 to 14.1, that Extent will reach the ſame Way from 12 to 84.6 Feet, the Content.



15.4 Baſe.
3.9 Half Perp.

1386
462

60.06 Product.

15.4 Baſe.	7.7 Half Baſe.
7.8 Perpend.	7.8 Perpendicular.
1232	616
1078	539
120.12	60.06 Product.
60.06 Half.	

Let ABC (*Fig.* 2.) be an oblique-angled Triangle given, whoſe Baſe is 15.4, and the Perpendicular 7.8; if 15.4 be multiply'd by 3.9 (half the Perpendicular), the Product will be 60.06 for the Area or ſuperficial Content: Or, if the Perpendicular 7.8 be multiply'd into half the Baſe 7.7, the Product will be 60.06 as before: Or, if 15.4, the Baſe, be multiply'd by the whole Perpendicular 7.8, the Product will be 120.12, which is the double Area; the Half thereof is 60.06 Feet, as before. See the Work.

By Scale and Compaſſes.

Extend the Compaſſes from 2 to 15.4, that Extent will reach from 7.8 to 60 Feet, the Content.

Demonſtration. If AD (*Fig.* 1.) be drawn parallel to BC, and DC parallel to AB; the Triangle ADC ſhall be equal to the given Triangle ABC. Hence the Parallelogram ABCD is double to the given Triangle; therefore half the Area of the Parallelogram

is

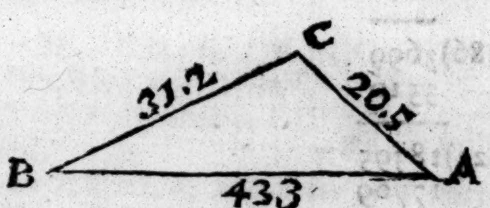
is the Area of the Triangle. In *Fig.* 2. the Parallelogram ABEF is alfo double to the Triangle ABC; for the Triangle ACF is equal to the Triangle ACD, and the Triangle BCE is equal to the Triangle BCD; therefore the Area of the Parallelogram is double to the Area of the given Triangle. Which was to be prov'd.

To find the Area of any plain Triangle, by having the three Sides given, without the Help of a Perpendicular.

The RULE.

Add the three Sides together, and take half that Sum: then fubtract each Side feverally from that half Sum. Which done, multiply that half Sum and the three Differences continually, and out of the laft Product extract the Square Root; which fquare Root fhall be the Area of the Triangle fought.

Example. Let ABC be a Triangle, whofe three Sides are as followeth, *viz.* AB 43.3, AC 20.5, and BC 31.2, the Area is requir'd.



Sides	43.3 31.2 20.5	4.2 16.3 27.0	Differences.
Sum	95.0		
Half	47.5		

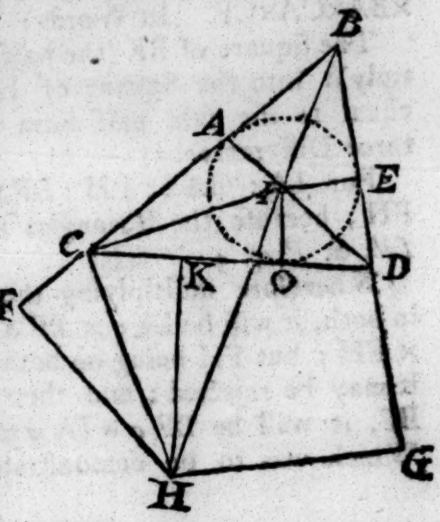
Area

Area 296.31

```
          47.5 The half Sum.
            27 Difference.
         -----
          3325
          950
         -----
        1282.5 Product.
          16.3 Difference.
        ------
         38475
        76950
       12825
      --------
      20904.75 Product.
           4.2 Difference.
      --------
      4180950
     8361900
     -------
     87799.9500(296.31
     4
     --
  49)477
     441
     ---
  586)3699
      3516
      ----
  5923)18395
       17769
       -----
 59261)62600
       59261
       -----
       3339 Remains.
```

Demon-

Demonſtratiom. In the Triangle BCD, I ſay, if from the half Sum of the Sides, you ſubtract each particular Side, and multiply the half Sum and the three Differences together continually, the ſquare Root of the Product ſhall be the Area of the Triangle.



Firſt, by the Lines BI, CI, and DI, biſect the three Angles, which Lines will all meet in the Point I; by which Lines the given Triangle is divided into three new Triangles, CBI, DCI, and BDI; the Perpendiculars of which new Triangles are the Lines AI, EI, and OI, being all equal to one another, becauſe the Point I is the Centre of the inſcribed Circle (by *Euclid*, *Lib.* 4. *Prop.* 4.): Wherefore to the Side BC join CF equal to DE, or DO; ſo ſhall BF be equal to half the Sum of the Sides, *viz.* $= \frac{1}{2}$ BC $+ \frac{1}{2}$ BD $+ \frac{1}{2}$ CD.

And BA $=$ BF $-$ CD; for CA $=$ CO and OD $=$ CF; therefore CD $=$ AF; and AC $=$ BF $-$ BD, for BE $=$ BA, and ED $=$ CF; therefore BD $=$ BA $+$ CF, and CF $=$ BF $-$ BC.

Then make CK $=$ CF, and draw the Perpendiculars FH, GH, and KH, and extend BI to H; becauſe the Angles FCK $+$ FHK are equal to two right Angles (for the Angles F and K are right Angles), equal alſo to FCK $+$ ACO (by *Euclid.* 1. 13.) and the Angles ACO $+$ AIO are equal to two right Angles; therefore the Quadrangles FCKH and AIOC are alike; and the Triangles CFH and AIC are alſo ſimilar. And the Triangles BAI and BFH are likewiſe ſimilar.

From

From this Explanation, I ſay, the Square of the Area of the given Triangle will be BF q × IA q = BF ×BA×CA×CF. In Words;

The Square of BF (the half Sum of the Sides) multiply'd into the Square of IA (=IF=IO) will be equal to the ſaid half Sum multiply'd into all the three Differences.

For IA : BA : : FH : BF; and IA : CF : : AC: FH; becauſe the Triangles are ſimilar. *By Euclid, Lib.* 6. *Prop.* 4.

Wherefore multiplying the Extremes and Means in both, it will be IA q × BF × FH = BA × CA × CF × FH; but FH being on both Sides of the Equation, it may be rejected; and then multiply each Part by BF, it will be BF q × IA q = BF × BA × CA × CF. Which was to be demonſtrated.

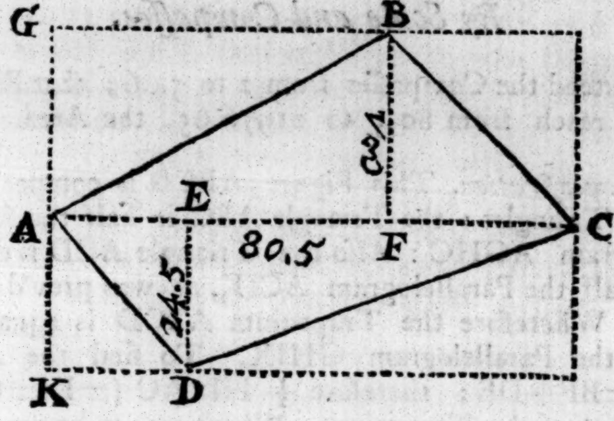
§ VI. *Of a* TRAPEZIUM.

A Trapezium is a Figure having 4 unequal Sides, and oblique Angles. To find the Area or ſuperficial Content thereof, this is

The RULE.

Add the two Perpendiculars together, and take half the Sum, and multiply that half Sum by the Diagonal, or multiply the whole Sum by half the Diagonal, the Product is the Area. Or you may find the Area's of the two Triangles, ABC and ACD, (by Section V.) and add thoſe two Area's together, the Sum ſhall be the Area of the Trapezium.

BF=30.1



BF = 30.1
DE = 24.5

Sum 54.6

Half 27.3
AC = 80.5

1365
2184

Area 2197.65

Let ABCD be a Trapezium given, the Diagonal whereof is 80.5, and the Perpendicular BF 30.1, and the Perpendicular DE 24.5; theſe two added together, the Sum is 54.6, the Half thereof is 27.3, which multiply'd by the Diagonal 80.5, the Product is 2197.65, which is the Area of the Trapezium; or if 40.25, half the Diagonal, be multiply'd by 54.6, the whole Sum of the Perpendiculars, the Product is 2197.65, the ſame as before.

By

By Scale and Compasses.

Extend the Compaſſes from 2 to 54.6; that Extent will reach from 80.5 to 21976.65, the Area.

Demonſtration. This Figure ABCD is compos'd of two Triangles; the Triangle ABC is half the Parallelogram AGHC: Alſo the Triangle ACD is equal to half the Parallelogram ACIK, as was prov'd Sect. V. Wherefore the Trapezium ABCD is equal to half the Parallelogram GHIK. To find the Area HI=BF+DE; therefore $\frac{1}{2}$ HI×AC (=KI=GH) =Area of the Trapezium. Which was to be prov'd.

§ VII. *Of Irregular* FIGURES.

IRregular Figures are all ſuch as have more Sides than four, and the Sides and Angles unequal. All ſuch Figures may be divided into as many Triangles as there are Sides, wanting two. To find the Area of ſuch Figures, they muſt be divided into Trapeziums and Triangles, by Lines drawn from one Angle to another; and ſo find the Area's of the Trapeziums and Triangles ſeverally, and then add all the Area's together; ſo will you have the Area of the whole Figure.

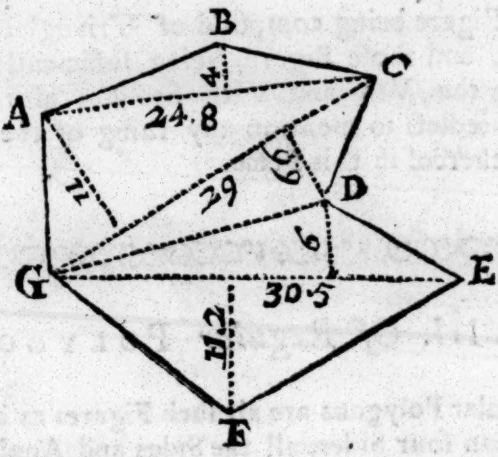
Let ABCDEFG be an irregular Figure given to be meaſur'd; firſt, draw the Lines AC and GD, and thereby divide the given Figure into two Trapeziums, ACGD and GDEF, and the Triangle ABC; of all which I find the Area's ſeverally.

Firſt, I multiply the Baſe AC by half the Perpendicular, and the Product is 49.6, the Area of the Triangle ABC.

Then

Then for the Trapezium ACGD, the two Perpendiculars, 11 and 6.6, added together, make 17.6; the half thereof is 8.8, multiply'd by 29, the Diagonal; the Product is 255.2, the Area of that Trapezium.

And for the Trapezium GDEF, the two Perpendiculars, 11.2 and 6, added together, make 17.2; the Half thereof is 8.6; which multiply'd by 30.5, the Diagonal, the Product is 262.3, the Area thereof. All these Area's added together, make 567.1, and so much is the Area of the whole irregular Figure. See the Work.



24 8 Base AC.
2 Half Perpendicular.

49.6 Area of ABC.

11 Perpendiculars.
6.6

17.6 Sum.
8.8 Half.
29 Diag. CG.

792
176

255.2 Area of ACGD.

11.2, 6	} Perpendiculars.	30.5, 8.6	
17.2	Sum.	1830	
		2440	
8.6	Half Sum.		
		262.30	Area of GDEF.
		255.2	Area of ACGD.
		49.6	Area of ABC.
		567.1	Sum of the Areas.

This Figure being compos'd of Triangles and Trapeziums, and thoſe Figures being ſufficiently demonſtrated in the Vth and VIth Sections aforegoing, it will be needleſs to mention any thing of the Demonſtration thereof in this Place.

§ VIII. *Of Regular* POLYGONS.

REgular Polygons are all ſuch Figures as have more than four Sides, all the Sidos and Angles thereof being equal. Polygons are denominated from the Number of their Sides and Angles.

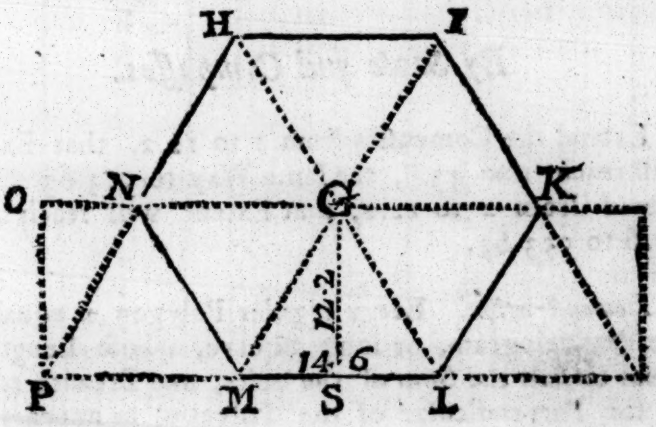
If the Figure conſiſts of	5	Equal Sides and Angles, it is called a regular	Pentagon.
	6		Hexagon.
	7		Heptagon.
	8		Octagon.
	9		Enneagon.
	10		Decagon.
	11		Endecagon.
	12		Dodecagon.

To

To find the Area or ſuperficial Content of any regular Polygon, this is

The RULE.

Multiply the whole Perimeter, or Sum of the Sides, by half the Perpendicular let fall from the Centre to the Middle of one of the Sides; or multiply the half Perimeter by the whole Perpendicular, and the Product is the Area.




```
 14.6
    3
 ----
 43.8 Half Sum of the Sides.
 ----
12.64 The Perpendicular.
 43.8 Half Sum.
-----
10112
 3792
5056
-------
553.632 Area.
```

```
  14.6
     6
  ----
  87.6 Sum of the Sides.
  6.32 Half Perpend.
  ----
  1752
 2628
5256
-------
553.632 Area.
```

Let HIKLMN be a regular Hexagon, each Side thereof being 14.6, the Sum of all the Sides is 87.6, the half Sum thereof is 43.8, which multiply'd by the Perpendicular GS 12.64, the Product is 553.63: Or, if 87.6, the whole Sum of the Sides, be multiply'd by half the Perpendicular 6.32, the Product is 553.63, the ſame as before, which is the Area of the given Hexagon.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 12.2, that Extent will reach from 43.8, the ſame Way to 553.63. Or, extend from 2 to 12.2, that Extent will reach from 87.6 to 553.63.

Demonſtration. Every regular Polygon is equal to the Parallelogram, or long Square, whoſe Length is equal to half the Sum of the Sides, and Breadth equal to the Perpendicular of the Polygon, as appears by the foregoing Figure; for the Hexagon HIKLMN is made up of ſix equilateral Triangles: And the Parallelogram OPQR is alſo compos'd of ſix equilateral Triangles, that is, five whole ones, and two Halves; therefore the Parallelogram is equal to the Hexagon.

A TABLE

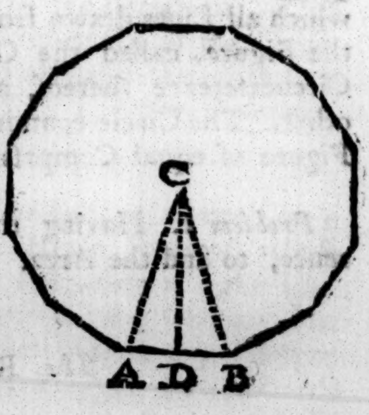
A TABLE *for the more ready finding the Area of a Polygon.*

Number of Sides.	Names.	Multipliers.
3	Trigon	.433013
4	Tetragon	1.000000
5	Pentagon	1.720477
6	Hexagon	2.598076
7	Heptagon	3.633959
8	Octagon	4.828427
9	Enneagon	6.181827
10	Decagon	7.694209
11	Endecagon	8.514250
12	Dodecagon	9.330125

Multiply the Square of the Side by the tabular Number, and the Product is the Area of the Polygon.

How to find thefe tabular Numbers.

Thefe Numbers are found by Trigonometry, thus; Find the Angle at the Centre of the Polygon by dividing 360 Degrees by the Number of Sides of the Polygon.



Example. Suppofe each Side of the Dodecagon annexed by 1, and the Area be required.

Divide 360 by 12 (the Number of Sides), and the Quotient is 30 Degrees for the Angle ACB; the Half thereof is 15, the Angle DCB, whoſe Complement to 90 Degrees is 75 Degrees, the Angle CBD: Then ſay,

As s,DCB 15 Degrees is	Co-ar. 0.587004
to .5 the Half-ſide DB. Log.	1.698970
ſo is s, CBD 73 Degrees.	9.984944
to the Perpendicular CD 1.866025	0.270918

Then 1.866025 multiply'd by .5 (the Half ſide), the Product is 9.330125 the Area of the Dodecagon requir'd.

§ IX. *Of a* CIRCLE.

A Circle is a plain Figure, contained under one Line, which is called a Circumference, unto which all Lines drawn from a Point in the Middle of the Figure, called the Centre, and falling upon the Circumference thereof, are all equal the one to the other. The Circle contains more Space than any plain Figure of equal Compaſs.

Problem 1. Having the Diameter and Circumference, to find the Area.

The RULE.

Every Circle is equal to a Parallelogram, whoſe Length is equal to half the Circumference, and the Breadth equal to half the Diameter; therefore multiply

ply half the Circumference by half the Diameter, and the Product is the Area of the Circle.

35.5	Half Cirumf.
11.3	Half Diameter.
1065	
355	
355	
401.15	Area.

Thus, if the Diameter of a Circle (that is, the Line drawn croſs the Circle through the Centre) be 22.6; and if the Circumference be 71, the Half of 71 is 35.5, and the Half of 22.6 is 11.3; which multiply'd together, the Product is 401.15, which is the Area of the Circle.

Demonſtration. Every Circle may be conceiv'd to be a Polygon of an infinite Number of Sides, and the Semidiameter muſt be equal to the Perpendicular of ſuch a Polygon, and the Circumference of the Circle equal to the Periphery of the Polygon; therefore half the Circumference, multiply'd by half the Diameter, gives the Area as aforeſaid.

Or (with *F. Ignat. Gaſton Pardies*), " Every Circle " is equal to a Rectangle-Triangle, one of whoſe " Legs is the Radius, and the other a right Line " equal to the Circumference of the Circle: For " ſuch a Triangle will be greater than any Polygon " inſcrib'd, and leſs than any Polygon circumſcrib'd " by the 24th, 25th, 26th, and 27th Articles of the " fourth Book of his Elements of Geometry); and " therefore muſt be equal to the Circle.

" For

" For (ſays he) ſhould it be greater than the
" Circle, be the Exceſs as little as it will, a Polygon
" may be circumſcrib'd, whoſe Difference, from the
" Circle, ſhall be yet leſs than the Difference between
" that Circle and the Rectangle-Triangle; and that
" that Polygon will be leſs than the Triangle, is ab-
" ſurd; and if it be ſaid, that this rectangled Triangle
" is leſs than the Circle, an inſcrib'd Polygon may be
" made, which ſhall be greater than that Triangle;
" which is impoſſible.

" This cannot but be admitted as a Principle,
" That if two determinate Quantities, A and B,
" are ſuch that if every imaginable Quantity,
" which is greater or leſs than A, is alſo greater or
" leſs than B, theſe two Quantities A and B muſt be
" equal.

" And this Principle being granted, which is in
" a manner ſelf-evident, it may directly be prov'd,
" that the Triangle (before mentioned) is equal to
" the Circle; becauſe every imaginable inſcrib'd
" Figure, which is leſs than the Circle, is alſo leſs
" than the Triangle; and every circumſcrib'd Figure,
" greater than the Circle, is alſo greater than the
" Triangle."

Problem 2. Having the Diameter of a Circle, to find the Cicrcumference.

As 7 to 22, ſo is the Diameter to the Circumference.

Or, as 113 to 355, ſo is the Diameter to the Circumference.

Or, as 1 to 3.141593, ſo is the Diameter to the Circumference.

Let the Diameter (as in the former Circle) be 22.6, this multiply'd by 22, and the Product is 497.2; which, divided by 7, gives 71.028 for the Circumference.

ference. Or (by the ſecond Proportion) if 22.6 be multiply'd by 355, the Product will be 8023; this divided by 11.3, the Quotient is 71, the Circumference. Or, (by the third Proportion) if 22.6 be multiply'd into 3.141593, the Product is 71.0000018 the Circumference; which two laſt Proportions are the moſt exact.

22.6	355
22	22.6
452	2130
452	710
	710
7)497.2(71.028	
	113)8023.01(71
	791
3.141593	113
22.6	113
18849558	...
6283186	
6283186	
71.0000018	

By Scale and Compaſſes.

Extend the Compaſſes from 7 to 22, or from 113 to 355, or from 1 to 3.14159; that Extent will reach from 22.6 to 71.

The Proportion of the Diameter of a Circle to the Circumference was never yet exactly found, notwithſtanding many eminent learned Men have labour'd very far therein; among which the excellent *Van Culen* hath hitherto outdone all, in his having calculated the ſaid Proportion to 36 Places of Decimals, which are engraven upon his Tomb-ſtone in *St. Peter*'s Church in *Leyden*; which Numbers are theſe;

Diameter,

" For (ſays he) ſhould it be greater than the " Circle, be the Exceſs as little as it will, a Polygon " may be circumſcrib'd, whoſe Difference, from the " Circle, ſhall be yet leſs than the Difference between " that Circle and the Rectangle-Triangle; and that " that Polygon will be leſs than the Triangle, is ab- " ſurd; and if it be ſaid, that this rectangled Triangle " is leſs than the Circle, an inſcrib'd Polygon may be " made, which ſhall be greater than that Triangle; " which is impoſſible.

" This cannot but be admitted as a Principle, " That if two determinate Quantities, A and B, " are ſuch that if every imaginable Quantity, " which is greater or leſs than A, is alſo greater or " leſs than B, theſe two Quantities A and B muſt be " equal.

" And this Principle being granted, which is in " a manner ſelf-evident, it may directly be prov'd, " that the Triangle (before mentioned) is equal to " the Circle; becauſe every imaginable inſcrib'd " Figure, which is leſs than the Circle, is alſo leſs " than the Triangle; and every circumſcrib'd Figure, " greater than the Circle, is alſo greater than the " Triangle."

Problem 2. Having the Diameter of a Circle, to find the Cicrcumference.

As 7 to 22, ſo is the Diameter to the Circumference.

Or, as 113 to 355, ſo is the Diameter to the Circumference.

Or, as 1 to 3.141593, ſo is the Diameter to the Circumference.

Let the Diameter (as in the former Circle) be 22.6, this multiply'd by 22, and the Product is 497.2; which, divided by 7, gives 71.028 for the Circumference.

ference. Or (by the ſecond Proportion) if 22.6 be multiply'd by 355, the Product will be 8023; this divided by 11.3, the Quotient is 71, the Circumference. Or, (by the third Proportion) if 22.6 be multiply'd into 3.141593, the Product is 71.0000018 the Circumference; which two laſt Proportions are the moſt exact.

```
   22.6
     22
   ----
    452
   452
   ----
7)497.2(71.028
```

```
        355
       22.6
       ----
       2130
       710
      710
      -----
113)8023.01(71
       791
       ---
       113
       113
       ---
       ...
```

```
  3 141593
      22.6
  --------
  18849558
  6283186
 6283186
 ---------
 71.0000018
```

By Scale and Compaſſes.

Extend the Compaſſes from 7 to 22, or from 113 to 355, or from 1 to 3.14159; that Extent will reach from 22.6 to 71.

The Proportion of the Diameter of a Circle to the Circumference was never yet exactly found, notwithſtanding many eminent learned Men have labour'd very far therein; among which the excellent *Van Culen* hath hitherto outdone all, in his having calculated the ſaid Proportion to 36 Places of Decimals, which are engraven upon his Tomb-ſtone in *St. Peter's* Church in *Leyden*; which Numbers are theſe;

Diameter,

Diameter.

1.00000.00000.00000.00000.00000.00000.00000

Circumference.

3.14159.26535.89793.23846.26433.83279.50288

Of which large Number theſe ſix Places, 3.14159, anſwering to the Diameter 1.00000, may be ſufficient; of the three Proportions, as 7 to 22, 113 to 355, and 1 to 3.14159, I ſhall leave my Reader to uſe which of them he pleaſes, but ſhall commend the laſt two as moſt exact, tho' the firſt be moſt in Uſe; but in the following Work I ſhall uſe ſometimes one of them, and ſometimes another, but for the moſt Part that of *Van Culen*, as being moſt exact.

Problem 3. Having the Circumference of a Circle, to find the Diameter.

As 1 is to .318309, ſo is the Circumference to the Diameter.

Or, as 355 to 113, ſo is the Circumference to the Diameter.

Or, as 22 to 7, ſo is the Circumference to the Diameter.

Let the Circumference be 71, (as in the former Circle) if .318309 be multiply'd by 71, (as by the firſt Proportion) the Product will be 22.599939 for the Diameter. Or, by the ſecond Proportion, 113 multiply'd by 71, the Product is 8023; which divided by 355, the Quotient will be 22.6 the Diameter. Or, by the third Proportion, 71 multiply'd by 7, the Product is 497; this divided by 22, the Quotient is 22.5909, the Diameter.

.318309

.318309		
71	113	71
	71	7
318309		
2228163	113	22)497(22.59
	791	44
22.599939		
	355)8023(22.6	57
	710	44
	923	130
	710	110
	2130	200
	2130	198
	...	2

Thus, by both the firſt Proportions, the Diameter is 22.6, but by the laſt it falls ſomething ſhort.

By Scale and Compaſſes.

Extend the Compaſſes from 3.14159 to 1, that Extent will reach from 71 to 22.6, which is the Diameter ſought.

Or, you may extend from 1 to .318309.

Or from 22 to 7.

Or from 355 to 113; the ſame will reach from 71 to 22.6, as before.

Note, That if the Circumference be 1, the Diameter will be .318309.

Problem 4. Having the Diameter of a Circle, to find the Area.

All Circles are in Proportion one to another, as are the Squares of their Diameters (by *Euc.* 12.2). Now, the Area of a Circle, whoſe Diameter is 1, will be .785398, according to *Van Culen*'s Proportion before

fore-mention'd; but for Practice .7854 will be ſufficient: Therefore,

As 1 (the Square of the Diameter 1) is to .7854, ſo is 510.76 (the Square of 22.6, the Diameter of the given Circle) to 401.15 (the Area of the given Circle): But,

According to *Metius*'s Proportion,

As 452 : 355 : : 510.76 : 401.15, the ſame as before.

But, if you uſe *Archimedes*'s Proportion, ſay,

As 14 : 11 : : 510.76 : 401.31; which Area is greater than by the two former Proportions; though in ſmall Circles this is near enough the Truth. See the Working of all theſe.

22.6 Diameter of the former Circle.
22.6

1356
452
452

510.76 The Square of the ſaid Diameter.

As 1 : .7854 : : 51076
.7854

204304
255380
408608
357532

401.150904 the Area.

By

By Scale and Compaſſes.

The Extent from 1 to 22.6, being twice turn'd over from .7854, will fall at the laſt upon 401.15, the Area.

```
 113          As 452 : 355 : : 510.76
   4                            355
 ---                          ------
 452                          255380
                             255380
                            153228
                           ---------
                       452)181319.80(401.15
                           1808
                           ----
                              519
                              452
                              ---
                               678
                               452
                               ---
                               2260
                               2260
                               ----
                               ....
```

As 14 : 11 :: 510.76
11

14)5618.36(401.31
56

18
14

43
42

16
14

2

Problem 5. Having the Circumference of a Circle, to find the Area.

Because the Diameters of Circles are proportional to their Circumferences; that is, as the Diameter of one Circle is to its Circumference; so is the Diameter of another Circle to its Circumference: Therefore the Areas of Circles are to one another, as the Squares of the Circumferences. And if the Circumference of a Circle be 1, the Area of that Circle will be .07958; then the Square of 1 is 1, and the Square of 71 (the Circumference of the former Circle) is 5041. Therefore it will be,

Sq. Cir. Area. Sq. Circumf.
As 1 : .07958 :: 5041
5041

7958
22 31832
4 397900

88 401.16278

Or

Or thus:

As 88 : 7 : : 5041

```
             5041
                7
88)35287(400.98 Area.
   352
   ---
     870
     792
     ---
355  780
  4  704
---  ---
1420  76
```

Or, As 1420 : 113 : : 5041 : 401.15 Area

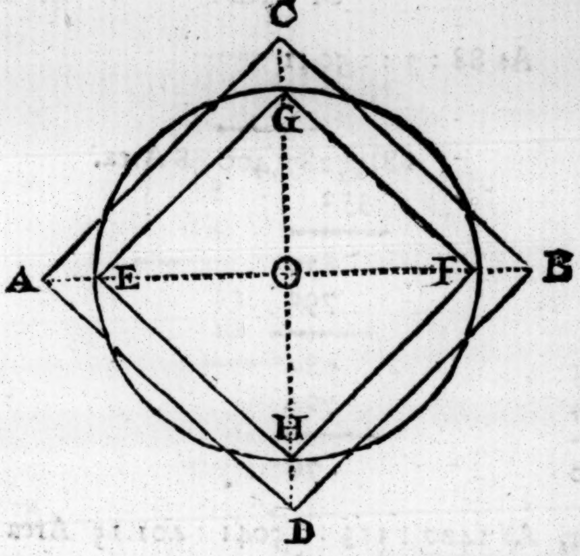
Problem 6. By having the Diameter, to find the Side of a Square that is equal in Area to that Circle.

If the Diameter of a Circle be 1, the Side of a Square equal thereunto will be .8862. Therefore,

As 1 : .8862 : : 22.6 (the Diameter)

```
  22.6
  -----
  53172
 17724
17724
------
```

To 20.02812 the Side of the Square AC.



Let the Diameter of the Circle EF or GH, be 22.6 (as before), to find the Side of the Square AC, A D, *&c.* If .8862 be multiply'd by 22.6 the Product is 20.02812, which is the Side of a Square, equal in Area to the Circle given; for if 20.02812 be multiply'd Square-wiſe, that is, by itſelf, it will produce 401.1255907344, which is nearly equal to the Area found in the laſt Problem.

You may find the Side of the Square equal, by extracting the Square Root out of the Area of the given Circle.

```
   · · ·
   401.15(20.0287295 Side of the Square.
   4
  ─────
4002)01.1500
        8004
      ──────
40048)349600
 ....  320384
       ──────
        29216
        28034
        ─────
         1182
          801
         ────
          381
          360
          ───
           21
           20
           ──
            1
```

N. B, By this Method of extracting the squareRoot of the Area, you may find the Side of a Square equal to any plain Figure, regular or irregular.

Problem 7. By having the Circumference, to find the Side of the Square equal.

If the Circumference of a Circle be 1, the Side of the Square equal will be .2821. Therefore,

As 1 : .2821 :: 71 (the Circumference).

```
     71
   ─────
    2821
  19747
  ──────
  20.0291 the Side of the Square.
```

Problem 8. Having the Diameter, to find the Side of a Square, which may be inscrib'd in that Circle.

If the Diameter of a Circle be 1, the Side of the Square inscrib'd will be .7071. Therefore,

As 1 : .7071 : : 22.6.

```
     .7071
      22.6
    ------
     42426
    14142
   14142
   -------
To 15.98046 the Side E G inscrib'd.
```

Or, if you square the Semidiameter, and double that Square, the square Root of the doubled Square will be the Side of the Square inscrib'd. For (by *Euclid.* 1.47) the Square of the Hypothenuse E G is equal to the Sum of the other two Legs, E O and O G.

```
  11.3 Semidiameter.
  11.3
  ----
   339
  113
 113
 ------
 127.69 the Squ. of EO, which double, because
      2                                (EO=OG.
 ------
 .  .  .
 255.38(15.98 Root, which is the Side of the Sq.
 1
 --
 25)155
    125
    ----
 309)3038
     2781
     -----
 3188)25700
      25504
      -----
        196
```

Problem

Problem 9. Having the Circumference, to find the Side of a Square which may be inſcrib'd.

If the Circumference be 1, the Side of the Square inſcrib'd will be .2251. Therefore,

As 1 : .2251 : : 71
71

2251
15757

15.9821 the Side of the Squ. EG.

Becauſe that in each of the four laſt Problems, *viz.* the 6th, 7th, 8th, and 9th, there is a Proportion laid down, it will be eaſy to work them with Scale and Compaſſes; for if you extend the Compaſſes from the firſt to the ſecond, that Extent will reach from the third to the fourth: As in the laſt Problem, where the Proportion is as 1 to .2251, ſo is 71 to the Side of the Square 15.9821. Here extend the Compaſſes from 1 to .2251; that Extent will reach from 71 to 15.98; and ſo of the reſt. But the fifth muſt be wrought like the 4th, thus; extend the Compaſſes from 1 to 71; that Extent turn'd over the ſame way from .07958, will fall, at the laſt, upon 401.15.

Problem 10. Having the Area, to find the Diameter.

If the Area of a Circle be 1, the Square of the Diameter thereof is 1.2732. Therefore,

Area

Area. Sq. Diam. Area.
As 1 : 1.2732 : : 401.15.

```
          401.15
        --------
          63660
         12732
        12732
      509280
      --------
      . .  . . .
      510.744180(22.599 the Diameter.
      4
      -----
   42)110
       84
      -----
  445)2674
      2225
      -----
 4509)44941
      40581
      ------
45189)436080
      406701
      ------
       29379
```

By Scale and Compasses.

Extend the Compasses from 1 to 1.2732 ; that Extent will reach from 401.15 to 510.74, *&c.* Then divide the Space between 1 and 510.74 into two equal Parts, and you'll find the middle Point at 22.6. Or you may divide the Space upon the Line of Numbers, between 401.15 and .7854, into two equal Parts, and one of those Parts will reach from 1 to 22.6, the Diameter sought.

Problem

Problem II. Having the Area, to find the Circumference.

If the Area of a Circle be 1, the Square of the Circumference will be 12.56637. Therefore,

```
Ar. Sq. Circumf.     Area.
As 1 : 12.56637 :: 401.15
          401.15
        --------
        6283185
       1256637
      1256637
     50265480
     -----------
     . . . . . . . Circumf.
     5040.99932550(70.9990 Root.
     49
     -----
  1409)14099
       12681
     -------
  14189)141893
        127701
     --------
 141989)1419225
        1277901
     ---------
1419989)14132450
        12779901
     ----------
        1352549
```

By Scale and Compasses.

Divide the Space between 401.15 and .07958, upon the Line, into two equal Parts; one of those Parts will reach from 1 to 71, the Circumference sought.

Problem

Problem 12. Having the Area, to find the Side of a Square inscrib'd.

If the Area of a Circle be 1, the Area of a Square inscrib'd within that Circle will be .6366. Therefore,

As 1 : 401.15 : : .6366
.6366

240690
240690
120345
240690

255.372090(15.98 Root, which is the Side of the
1 (Square sought.

25)155
125

309)3037
2781

3188)25620
25504

11690

The same Reason may be given for the last Proportion, that was given before for the Proportion of Circles to the Squares of their Diameters and Circumferences; for not only the Squares of the Diameters and Circumferences are in Proportion to the Circles they belong to, but also all Figures inscribed or circumscrib'd, have the Squares of their like Sides proportional to the Circles they are inscrib'd in, or circumscrib'd about; and also to the Figures themselves: The Square of any Side of one Figure is to the Area of that

that Figure, as the Square of the like Side of another similar Figure is to the Area thereof: as you may find prov'd at large in *Euclid, Sturmius, Mathesis Enucleata*, and other Authors; but will be too large to insert in this Place.

By Scale and Compasses.

Extend the Compasses from 1 to 401.15, that Extent will reach from .6366 to 255.37; the half Space between that and 1 is at 15.98, the Side of the Square.

Problem 13. Having the Side of a Square, to find the Diameter of the circumscribing Circle.

If the Side of a Square be 1, the Diameter of a Circle that will circumscribe that Square, will be 1.4142. Therefore,

As 1 : 1.4142 : : 15.98

```
  15.98
 ------
 113136
127278
70710
14142
---------
22.598916 the Diameter sought.
```

By Scale and Compasses.

Extend the Compasses from 1 to 1.4142, and that Extent will reach from 15.98 to 22.6, the Diameter sought.

Problem

Problem 14. Having the Side of a Square, to find the Diameter of a Circle equal.

If the Side of a Square be 1, the Diameter of a Circle equal thereunto will be 1.128. Therefore,

Side Diam. Side of a Square.
As 2 : 1.128 : : 20.0291.
1.128

1602328
400582
200291
200291

22.5928248 Diameter.

By Scale and Compasses.

Extend the Compasses from 1 to 1.128; that Extent will reach from 20.0291, (the Side of the Square given) to 22.6, the Diameter of the Circle sought.

Problem 15. Having the Side of a Square, to find the Circumference of the circumscribing Circle.

If the Side of a Square be 1, the Circumference of a Circle that will encompass that Square will be 4.443. Therefore,

Side Sq. Circum. Side Sq.
As 1 : 4.443 : : 15.98
15.98

35544
39987
22215
4443

70.99914 the Circumf.

By Scale and Compasses.

Extend the Compasses from 1 to 4.443, that Extent will reach from 15.98 to 71, the Circumference.

Problem

5

Problem 16. Having the Side of a Square, to find the Circumference of a Circle that will be equal thereunto.

If the Side of the Square be 1, the Circumference of a Circle that will be equal thereunto, ſhall be 3.545. Then,

$$
\begin{array}{r}
\text{As } 1 : 3.545 :: 20.0291 \\
3.545 \\
\hline
1001455 \\
801164 \\
1001455 \\
600873 \\
\hline
71.0031595
\end{array}
$$

71.0031595 the Circumference.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 3.545, that Extent will reach from 20.0291 to 71, the Circumference ſought.

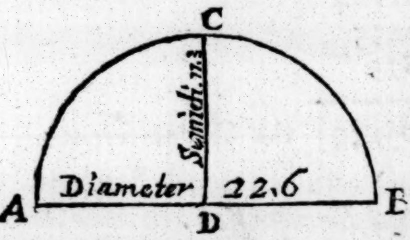
In ſeveral of the foregoing Problems, where the Diameter and Circumference is requir'd, the Anſwers are not exactly the ſame as the Diameter and Circumference of the given Circle, but are ſometimes too much, and ſometimes too little, as in the two laſt Problems, where the Anſwers in each ſhould be 71, the one being too much, and the other too little. The Reaſon of this is, the ſmall Defect that happens to be in the Decimal Fractions, they being ſometimes too great, and ſometimes too little; yet the Defect is ſo ſmall, that it is needleſs to calculate them to more Exactneſs.

§ X. *Of a* SEMICIRCLE.

TO find the Area of the Semicircle, this is

The RULE.

Multiply the fourth Part of the Circumference of the whole Circle (that is, half the Arch-line) by the Semidiameter, the Product is the Area.



Let ABC be a Semicircle, whoſe Diameter 22.6, and the half Circumference, or Arch-line, A C B, is 35.5. the Half thereof is 17.75. which multiply by the Semidiameter 11.3, and the Product is 200.575, the Area of the Semicircle.

```
17.75 the half Arch-line.
 11.3 the Semidiameter.
------
 5325
 1775
1775
------
200.575 the Area of the Semicircle.
```

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 11.3; that Extent will reach from 17.75 to 200.575, the Area.

If only the Diameter of the Semicircle be given, you may ſay, by the *Rule of Three*;

As 1 is to .3927, ſo is the Square of the Diameter to the Area.

By

By Scale and Compasses.

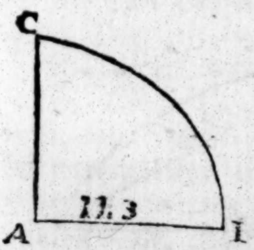
Extend the Compasses from 1 to the Diameter 22.6; that Extent turn'd twice from .3927, will reach, at the last, to 200.575.

§ XI. *Of a* QUADRANT.

TO find the Area of a Quadrant, or fourth Part of a Circle, this is

The RULE.

Multiply half the Arch-line of the Quadrant (that is, the eighth Part of the Circumference of the whole Circle), by the Semidiameter, and the Product is the Area of the Quadrant.



Let ABC be a Quadrant, or fourth Part of a Circle, whose Radius, or Semidiameter, is 11.3, and the half Arch-line 8.875; these multiply'd together, the Product is 100.2875 for the Area.

These are the Rules and Ways commonly given for finding the Area of a Semicircle and Quadrant; but, I think, it is as good a Way, to find the Area of the whole Circle, and then take half that Area for the Semicircle, and a fourth Part for the Quadrant.

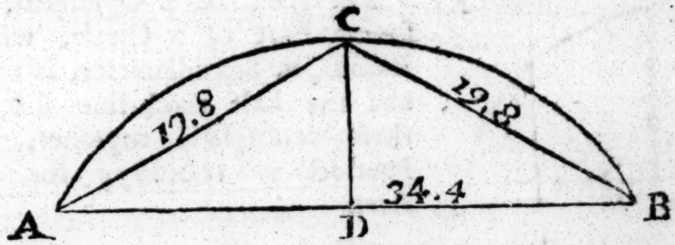
Before I proceed to shew how to find the Area of the Sector, and Segment of a Circle, I shall shew how to find the Length of the Arch-line, both *Geometrically* and *Arithmetically*.

To find the Length of the Arch-line *Geometrically.*

Divide the Chord-line A B into four equal Parts, and ſet one of theſe Parts from B to C, and draw a Line from C to three of thoſe Parts at D; ſo ſhall CD be equal to half the Arch-line ACB.

To find the Length of the Arch-line *Arithmetically.*

Multiply the Chord of half the Segment AC or CB by 8, and from the Product ſubtract the Chord of the whole Segment AB, and divide the Remainder by 3; the Quotient is the Arch-line ACB ſought.



19.8 AC
8

158.4
34.4 AB

3)124

Arch-line 41.333

Another

Another Way.

From the double Chord of half the Segment's Arch, ſubtract the Chord of the Segment, one third Part of the Difference added to the double Chord of half the Segment's Arch, the Sum is the Arch-line of the whole Segment.

Thus, if AC 19.8 be doubled, it makes 39.6; from which if you ſubtract 34.4, the Remainder is 5.2, which divided by 3, the Quotient is 1.733; this added to 39.6, (the double Chord of the half Segment) the Sum is 41.333. So if the Arch-line ACB was ſtretch'd out ſtrait, it would then contain 41.333 ſuch Parts as the Chord AB contains 34.4 of the like Parts.

Theſe two Rules may very eaſily be prov'd out of the Table of natural Sines; thus,

Suppoſe (in the former Figure) the Arch ACB to contain 120 Degrees; the natural Sine of half, *viz.* of 60 Degrees, is 86602, which being doubled, is 173204, which is the Chord of the whole 120 Degrees, that is, AB. Then, to find the Chord of the half Arch, *viz.* AC 60 Degrees, the Half of it 30 Degrees, the natural Sine thereof is 50000; which, doubled, makes 100000 for the Chord AC; then, according to the firſt Rule, multiply 100000 by 8, the Product is 800000; from which ſubtract 173204, (the Chord AB and the Remainder is 626796; which divide by 3, the Quotient is 208932, which is the Length of the Arch-line ACB, according to the firſt Rule. Now let us examine how near this comes to the true Quantity of the Arch propos'd. If the Radius or Semidiameter of a Circle be 100000, (as it is in the Table of Sines) then the Circumference will be 628318; and becauſe 120 Degrees is the third Part of the Circle, take the third Part of 628318 is 209439, which is the true Quantity of the Arch ACB in ſuch Parts as the

the Semidiameter contains 100000, and differs from that before found 507, which is a thing inconſiderable in *Practical Menſuration.* The latter of the foregoing Rules agrees exactly with the former, and therefore the Difference will be the ſame as above; either of the Rules gives the Quantity of the Arch-line too little, and the greater the Arch, the greater the Error. If you know the Degrees that are contain'd in the Segment's Arch, and would have the Arch-line very exactly, you may reaſon thus by the *Rule of Three.*

As the Circle's Periphery in Degrees, is to its Periphery in equal Parts; ſo is the Arch in Degrees and decimal Parts, to the ſame Arch in equal Parts.

Suppoſe the Circumference of a Circle be 71, and ſuppoſe the Arch to contain 52 Degrees, 15 Minutes, (the Decimal of 15 Minutes is .25) then ſay,

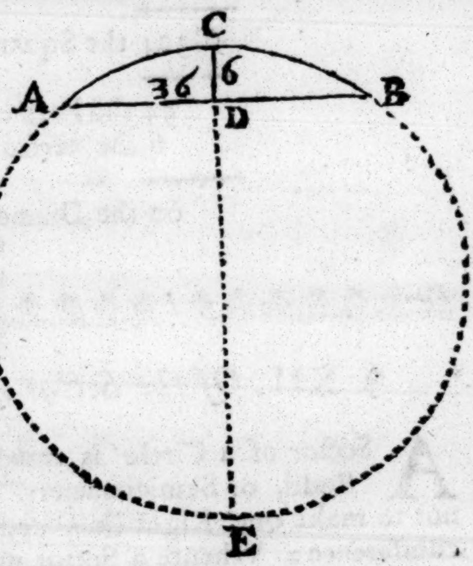
```
   Deg. Parts   Deg.
As 360 : 71 :: 52.25
                  71
               -----
                5225
               36575
               -----
    36|0)370|9.75(10.305 fere
         36
         ---
          109
          108
          ---
          175
```

So the 52 Degrees 15 Minutes wil contain 10.305 of ſuch Parts as the Circumference contains 71.

Thus have I ſhew'd ſeveral Ways of finding the Meaſure of the Curve-line of any Part of a Circle very near the Truth. The next thing I ſhall ſhew, is,

How

How to find the Diameter of a Circle by having the Chord and verſed Sine of the Segment Arithmetically.



Becauſe the Chord A B cuts the Diameter E C at right Angles, therefore the Semichord A D or D B is a mean proportional Line between the Parts of the Diameter C D and D E (by *Euc.* 6. 13.); therefore, if you ſquare the Semichord AD or DB, and divide that Square by the verſed Sine CD, the Quotient will be the Part of the Diameter wanting; to which add the given verſed Sine CD, ond the Sum is the Diameter ſought.

Example. Let ACB be a Segment given, whoſe Chord AB is 36, and the verſed Sine CD 6; half 36 is 18, which ſquar'd, makes 324; this divided by 6, the Quotient is 54; to which add 6, the Sum is 60, the Diameter of the Circle CE.

18 half the Chord.
18
———
144
18
———
6)324 the Square of AD.
———
54 the Part wanting DE.
6 the versed Sine CD add.
———
60 the Diameter CE.

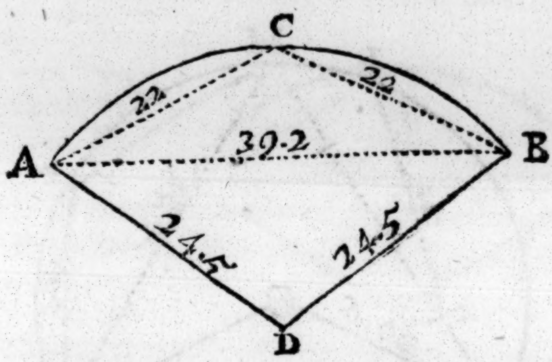
§ XII. *Of the Sector of a* Circle.

A Sector of a Circle is comprehended under two Radii, or Semidiameters, which are suppos'd not to make one Right-line, and a Part of the Circumference: Whence a Sector may be either less or greater than a Semicircle. To find the Area or superficial Content thereof, this is

The RULE.

Multiply half the Arch-line by the Semidiameter, and the Product is the Area.

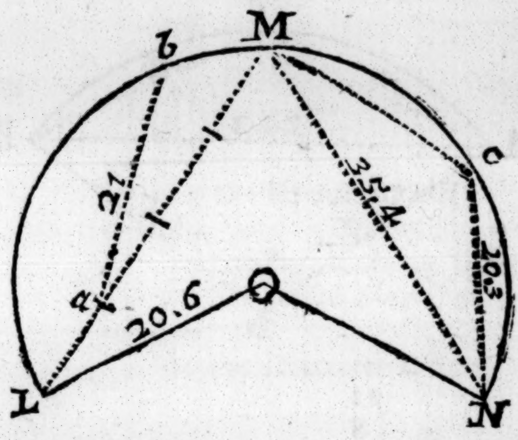
Let ADBC be the Sector of a Circle given, whose Semidiameter AD or BD is 24.5, and the Arch-line ACB (by the first Rule, *Pag.* 112.) I find to be 45.6; the Half thereof 22.8, being multiply'd by 24.5, (the Semidiameter) the Product is 558.6, which is the Area of the Sector ACBD.




```
     22
      8
    ----
    176
    39.2 Subtrahend.
   -----
  3)136.8
   -----
   45.6 Arch-line.
   ----
   22.8 half.
   24.5 Semidiameter.
   -----
   1140
   912
  456
  ------
  558.60 the Area.
```

Again. Let LMNO be a Sector greater than a Semicircle, whoſe Semidiameter LO or NO is 20.6, and the Line *b a* equal to a fourth Part of the Arch-line L *b* MN 21, the Double whereof is 42, equal to the Arch-line L*b*M or M *c* N; or by the Arithmetical Rule, *Pag.* 112 the ſaid Arch is found to be 42.333, which multiply'd by 20.6, the Semidiameter, makes 872.0598 for the Area of the Sector LMNO.

See the following Work.



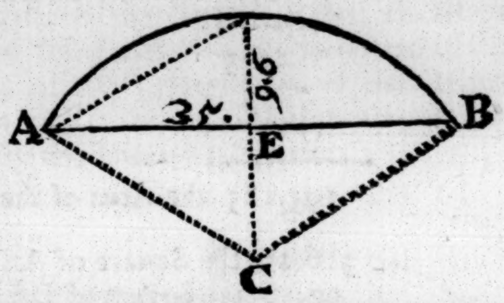
20.3 Chord N *c*	42.333
8	20.6
162 4	253998
35.4 Chord MN.	84666
3)12.70	872.0598 Area.
42.333 Arch-line.	

§ XIII. *Of the Segment of a* Circle.

A Segment of a Circle is a Part terminated by a Right-line less than the Diameter, call'd a Chord, and by a Part of the Circumference.

To find the Area of the Segment of a Circle, you must, first, find the Centre of the whole Circle, and draw the two Semidiameters, thereby completing the Sector, as in the following Figure. Then (by the last Section) find the Area of the whole Sector CADBC, and then (by Sect. 5.) find the Area of the Triangle ABC, and subtract the Area of the Triangle out of the Area of the Sector, the Remainder is the Area of the Segment.

Other-



Otherwiſe you may, without deſcribing the Figure, find the Semidiameter of the Circle by the Arithmetical Rule, *pag.* 113. and by the Arithmetical Rule, *pag.* 112. find the Arch-line; then multiply half the Arch-line by the Semidiameter; ſo have you the Area of the Sector. Then ſubtract the verſed Sine from the Semidiameter, the Remainder is the Perpendicular of the Triangle; and multiply the half Chord by the Perpendicular, the Product is the Area of the Triangle. Then ſubtract the Area of the Triangle from the Area of the Sector, and the Remainder is the Area of the Segment. See the Work.

2)35=AB

17.5
17.5

875
1225
175

9.6)306.25(31.9
9.6 add

182
865 41.5 the Diameter of the Circle.

1 20.75 the Semidiameter.
9.6 D E Subtrahend.

11.15 remains the Perpendicular EC.

11.15 the Perpendicular EC.
17.5 half the Chord AE or EB.

5575
7805
1115

195.125 the Area of the Triangle.

306.25 the Square of AE.
92.16 the Square of DE the verſed Sine.

398.41 Sum.

The ſquare Root thereof is 19.96 the Chord AD.
8

159.68
Sub. 35 the Chord AB.

3)124.6

2)41.56 the Arch-Line.

20.78 half.
20.75 Semidiameter.

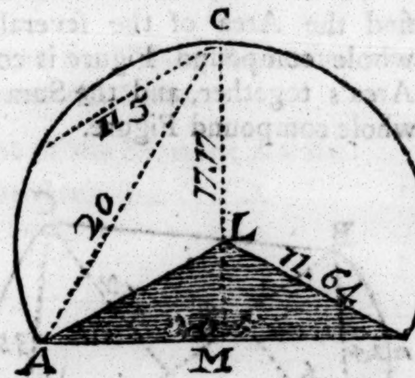
10390
14546
4156

From 431.1850 Area of the Sect.
Subtract 195.125 Area of the Trian

Remains 236.060 Area of the Segm.

Again,

Again, let MACBM be a Segment greater than a Semicircle; observe the former Rules, in all respects, as in the last Example; only, instead of subtracting the Area of the Triangle out of the Area of the Sector, here you must add it thereunto, as may plainly appear by the following Figure.



11.5
8

92.0
20

3)72

24 Half Arch-line.

11.64 Semi-diam.
24

4656
2328

279.36 Area of the Sector LACBL.

17.17
11.64

5.53 LM.

10.25 half the Base MA.
5.53 the Perpendicular LM.

3075
5125
5125

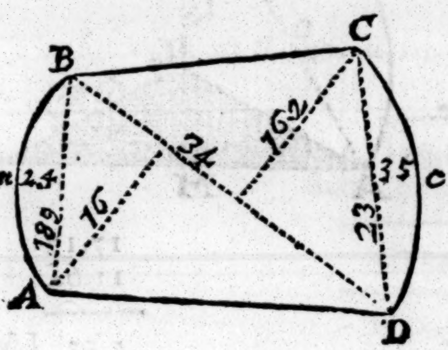
56.6825 the Area of the Triangle ALM.
279.36 the Area of the Sector add.

336.0425 the Area of the Segment sought.

§ XIV. *Of Compound* FIGURES.

MIX'D or compound Figures are ſuch as are compos'd of rectilineal and curvilineal Figures together.

To find the Area of ſuch mix'd Figures, you muſt find the Area of the ſeveral Figures of which the whole compound Figure is compoſed, and add all the Area's together, and the Sum will be the Area of the whole compound Figure.



16.2 }
16 } add.

32.2 Sum.

16.1 half.
34 Diagonal.

644
483

547.14 Area of the (Trapezium.

10.236 half the Arch-line A a B.
14.83 Semidiameter of the Arch A B.

30708
81888
40944
10236

151.79988 Area of the Sector.

From

From 14.83 Semidiameter.
Subtract 3.4 verſed Sine.

11.43 Perpend. of the Triangle.
9.45 half the Chord AB.

5715
4572
10287

108.0135 the Area of the Triang. ſubtracted from
151.7999 the Area of the Sector.

43.7864 the Area of the Segment A a BA.

12.19 half the Arch-line C c D.
20.64 Semidiameter.

4876
7314
24380

251.6016 the Area of the Sector.

From 20.64 the Semidiameter,
Subtract 3.5 verſed Sine.

Remaind. 17.14 Perpendicular of the Triangle.
11.5 half the Chord DC.

8570
1714
1714

Sub. 197.110 Area of the Triangle.
From 251.602 Area of the Sector.

Rem. 54.492 the Area of the Segment Cc DC.
43.786 the Area of the Segment Aa BA.
547.4 The Area of the Trapezium.

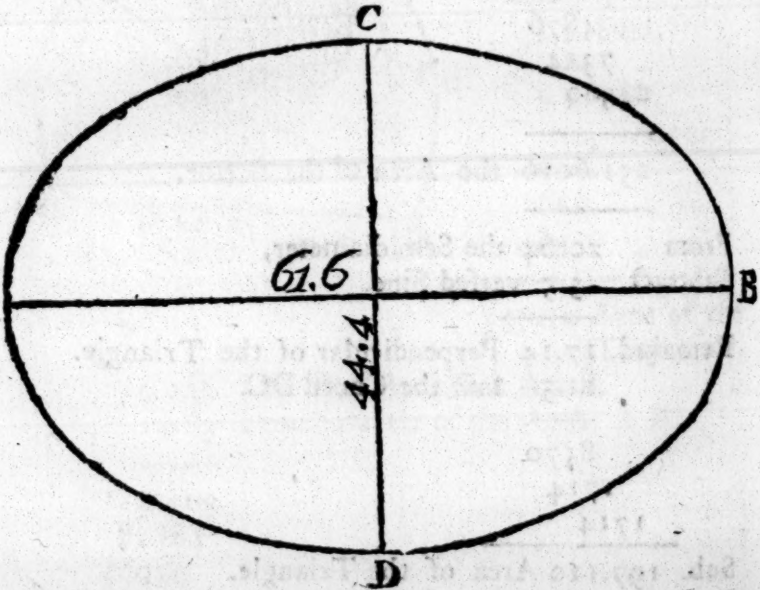
Sum 645.678 the Area of the Whole.

§ XV. *Of an* Ellipsis.

AN Ellipſis, or Oval, is a Figure bounded by a regular curve Line, returning into itſelf; but of its two Diameters, cutting each other in the Centre, one is longer than the other, in which it differs from the Circle. To find the Area thereof, this is

The RULE.

Multiply the tranſverſe Diameter by the Conjugate, and multiply that Product by .7854, this laſt Product is the Area of the Oval.



61.6 the tranſverſe Diameter.
44.4 the conjugate Diameter.

2464
2464
2464

2735.04 the Rectangle.
.7854 the Area of Unity.

1094016
1367520
2188032
1914528

2148.100416 the Area of the Oval.

Demonſtration. If you circumſcribe any Ellipſis with a Circle, and ſuppoſe an infinite Number of Chord-lines drawn therein, all parallel to the conjugate Diameter, as thoſe in the following Figure, then it will be,

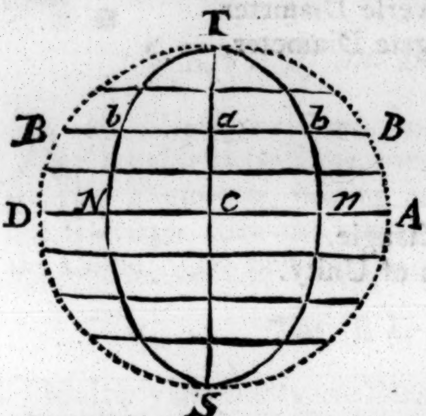
As DA, the Diameter of the Circle, is to N n, the conjugate Diameter of the Ellipſis; ſo is B a B, any Chord in the Circle, to b a b, its reſpective Ordinate in the Ellipſis.

For, according to the Property of the Circle,

it is	1	a S × T a = $\square$ B a.
And		by the Property of the Ellipſis,
it is	2	$\square$ T C : $\square$ N C : : a S × T a : $\square$ b a.
1, 2,	3	$\square$ T C : $\square$ N C : : $\square$ B a : $\square$ b a.
3, hence	4	T C : N C : : B a : b a.
Conſeq.	5	T C : 2 N C : : 2 B a : 2 b a.
That is	6	D A : N n : : B a B : b a b.

But the Sum of an infinite Series of ſuch Chords as B a b, do conſtitute the Area of the Circle. And the Sum of the like Series of their reſpective Ordinates, as b a b, do conſtitute the Area of the Ellipſis:

 There-



Therefore, T S : N n : : Circle's Area : the Ellipsis Area. But T S : N n : : □ TS : TS × Nn; whence it follows that,

□ T S : Circle's Area : : T S × Nn : Ellipsis Area.

Consequently, As 1 is to .7854; so is the Rectangle, or Product of the transverse and conjugate Diameter of any Ellipsis to its Area.

Hence it is easy to conceive, that the square Root of the Product of the transverse and conjugate Diameters will be the Diameter of a Circle equal to the Ellipsis.

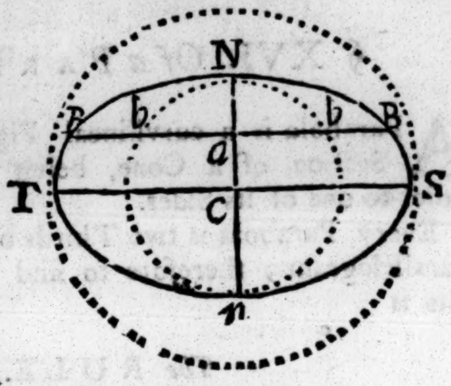
Hence also all Segments of an Ellipsis, and its circumscribing Circle, (whose Bases are parallel to the conjugate Diameter, and of the same Height) are in Proportion one to another as their Bases are; that is,

B a B : b a b : : Area Segment BTB : Area Segment b T b.

Or, TS : N n : : Area Segment BTB : Area Segment b T b.

The Area of every Ellipsis is a mean Proportional between the Areas of its circumscribing and inscrib'd Circles.

The



The Truth of this may easily be reduc'd from the last, for 'tis already prov'd, that □ TS : TS × Nn : : circumscribing Circle's Area : Ellipsis Area.

But □ TS : TS × Nn : : TS × Nn : □ Nn. Therefore Ellipsis Area : inscrib'd Circle's Area : TS× Nn : □ Nn.

Example, Let TS=36, and Nn=18.4.
Then □ TS=1296, and □ Nn=338.56.
Then 1296 × .7854=1017.8784 great Circle's Area;
And 338 56×.7854=265.905,&c. lesser Circle's Area;
And 36× 18.4=662.4 × .7854=520.24896, which is the Area of the Ellipsis; then it will be,

1017.878 : 520.24896 : : 520.24896 : 265.905024.

That is, As the great Circle's Area is to the Area of the Ellipsis; so is the Area of the Ellipsis to the Area of the lesser Circle.

From hence it follows, that all Segments of an Ellipsis, and its inscrib'd Circle, (whose Bases are parallel to the transverse Diameter, and have the same Height) are in Proportion one to another as the Area of the Ellipsis and Circle are.

That is, As the Area of the Circle is to the Area of the Ellipsis, so is the Segment b N b : to the Segment BNB.

Or, Nn : TS : : Area Segment b N b : Area Segment BNB.

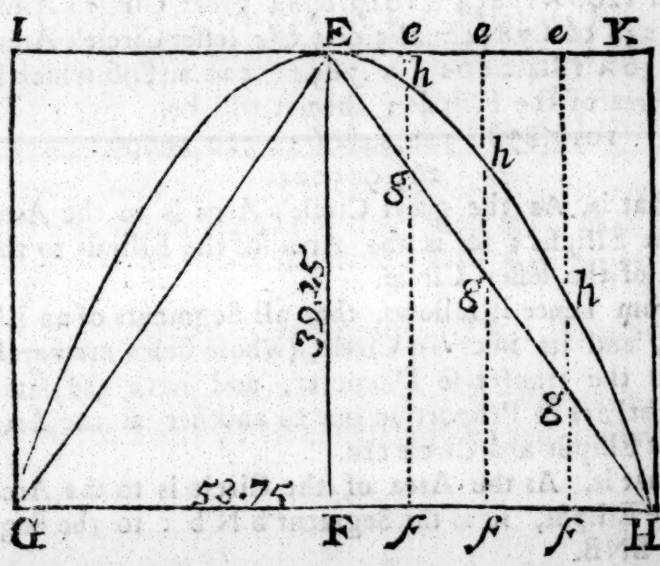
§ XVI. *Of a* PARABOLA.

A Parabola is a curvilineal Figure, made by the Section of a Cone, being cut by a Plane parallel to one of its Sides.

Every Parabola is two Thirds of its circumſcribing Parallelogram; therefore to find the Area thereof, this is

The RULE.

Multiply the Baſe, or greateſt Ordinate, by the perpendicular Height, and multiply that Product by 2, and divide the laſt Product by 3, the Quotient will be the Area of the Parabola.



53·75

$$\begin{array}{r}
53.75 \text{ the Ordinate GH.} \\
39.25 \text{ the Perpendicular EF.} \\
\hline
26875 \\
10750 \\
48375 \\
16125 \\
\hline
2109.9875 \\
2 \\
\hline
3)4219.3750 \\
\hline
1406.4583 \text{ the Area.}
\end{array}$$

Demonſtration. Let FH, the Semi-ordinate, be divided into four equal Parts, or into 8.16, &c. and through the Diviſions draw Lines, as e f, e f, &c. parallel to the Axis EF. Suppoſe alſo EF to be 4.

Then, I ſay, the Parabolic Sphere Eh HF is to the Parallelogram EKFH as 2 to 3; but to the Triangle EFH as 4 to 3.

For, firſt, g f, g f, g f, &c, are in continual arithmetical Proportion from the Nature of plain Triangles.

Secondly, f e : g e : : g e : h e ; but h e in the Axis EF = o. and in the firſt Parallel e f muſt be equal to $\frac{1}{4}$, in the next e f muſt be equal to $\frac{4}{4}$, in the third to $\frac{9}{4}$, and ſo on, in a duplicate arithmetical Progreſſion.

For ef (=4) : g e (=1) : : g e (=1) : e h (=$\frac{1}{4}$) And the ſecond e f (=4) : e g (=2) : : e g (=2) e h (=$\frac{4}{4}$), &c. And thus it will be, if the Lines F f, f f, &c. be again biſected, &c. *ad infinitum*, ſo that all the Indiviſibles of the trilinear Space EKHh E will be in a duplicate arithmetical Progreſſion increaſing. But the Sum of a Rank of ſuch Terms is ſubtriple to a Rank of as many equal to the greateſt (by Lemma 3); wherefore the whole trilinear Space EKHhE is to the Paralellogram as 1 to 3; and, conſequently, the remaining parabolic Space muſt be to it as 2 to 3; which was to be proved.

And

And ſince the Triangle FEH is to the Parallelogram as 1 to 2, it muſt be to the Parabola as $1\frac{1}{2}$ to 2. or as 3 to 4; which was to be prov'd.

Before I proceed to the Menſuration of ſolid Bodies, I will lay down ſuch Lemma's as will be neceſſary to facilitate the Demonſtration of all ſuch Solids.

LEMMA I.

In any Series of equal Numbers (repreſenting Lines or other Quantities) as 1, 1, 1, 1, *&c.* or 2, 2, 2, 2, *&c.* or 3, 3, 3, 3, *&c.* if one of the Terms be multiply'd into the Number of Terms, the Product will be the Sum of all the Terms in the Series.

LEMMA II.

If a Series of Numbers, in arithmetical Progreſſions, begin with a Cypher, and the common Difference be 1, as 0, 1, 2, 3, *&c.* (repreſenting a Series of Lines or Roots beginning with a Point) if the laſt Term be multiply'd into the Number of Terms, the Product will be double the Sum of all the Series.

That is, putting L = the laſt Term, N = the Number of Terms, and S = the Sum of all the Series; then will NL = 2S; conſequently, $\frac{1}{2}$ NL = S, *viz.* One-half of ſo many times the greateſt Term as there are Numbers of Terms in the Series.

$$\text{Thus}\begin{cases} 0+1+2+3+4=10 \text{ the Sum } = \frac{1}{2}\text{ NL.} \\ 4+4+4+4+4=20=\text{NL.} \end{cases}$$

LEMMA III.

If a Series of Squares, whoſe Sides or Roots are in arithmetical Progreſſion, beginning with a Cypher, *&c.* be infinitely continued, the laſt Term, being multiply'd into the Number of Terms, will be triple to

to the Sum of all the Series, *viz.* NLL$=$3S; or, $\frac{1}{3}$ NLL$=$S.

That is, the Sum of ſuch a Series will be One-third of the laſt or greateſt Term, ſo many times repeated as there are Numbers of Terms in their Series.

Inſtances in ſquare Numbers.

$$1\left\{\frac{0+1+4}{4+4+4}=\frac{5}{12}=\frac{1}{3}+\frac{1}{12}.\right.$$

$$2\left\{\frac{0+1+4+9}{9+9+9+9}=\frac{14}{36}=\frac{7}{18}=\frac{1}{3}+\frac{1}{18}.\right.$$

$$3\left\{\frac{0+1+4+9+16}{16+16+16+16+16}=\frac{30}{80}=\frac{3}{8}=\frac{9}{24}=\frac{1}{3}+\frac{1}{24}.\right.$$

(*&c.*

From theſe Inſtances it is evident, that as the Number of Terms in the Series do increaſe, the Fraction or Exceſs above $\frac{1}{3}$ does increaſe, the ſaid Exceſs always being $\frac{1}{6N-6}$; which, if we ſuppoſe the Series to be infinitely continued, will quite vaniſh, and become nothing at all.

LEMMA IV.

If a Series of Cubes, whoſe Roots are in arithmetical Progreſſion, beginning with a Cypher, *&c.* (as above) be infinitely continued, the Sum of all the Series will be $\frac{1}{4}$ NLLL$=$S.

That is, One-fourth of the laſt Term ſo many times repeated as there are Numbers of Terms.

Inſtances

Inſtances in cube Numbers.

If 0, 1, 2, 3, 4, 5, *&c.* be the Roots of the Cubes,

$$1 \left\{ \frac{0+1+8+27}{27+27+27+27} = \frac{36}{108} = \frac{4}{12} = \frac{1}{4} + \frac{1}{12}. \right.$$

$$2 \left\{ \frac{0+1+8+27+64}{64+64+64+64+64} = \frac{100}{320} = \frac{10}{32} = \frac{5}{16} = \frac{1}{4} + \frac{1}{16}. \right.$$

$$3 \left\{ \frac{0+1+8+27+64+125}{125+125+125+125+125+125} = \frac{125}{750} = \frac{45}{150} = 20 = \frac{1}{4} + \frac{1}{20}. \right.$$

From theſe Examples it plainly appears, that as the Number of Terms in the Series increaſes, the Fraction or Exceſs above $\frac{1}{4}$ decreaſes, the Exceſs being always $\frac{1}{4N-4}$; which, if we ſuppoſe the Series to be infinitely continued, will become infinitely ſmall or nothing.

LEMMA V.

If a Series of Biquadrates, whoſe Roots are in arithmetical Propreſſion, beginning with a Cypher, *&c.* as beſore, be infinitely continued, the Sum of all the Terms in ſuch a Series will be $\frac{1}{5}$ NLLLL.

The Truth of this may be manifeſt by the like Proceſs as in the foregoing Lemma's, and ſo on for higher Powers.

LEMMA VI.

The Sum of an infinite Progreſſion, whoſe greateſt Term is a ſquare Number, the others decreaſing by odd Numbers, *viz.* 1, 3, 5, *&c.* is ſn ſubſeſquialteran Proportion oſ the Sum of the like Number of equal Terms, that is, as 2 to 3.

Inſtances in ſuch Progreſſions.

1 $\left\{\frac{9+8+5}{9+9+9}=\frac{22}{27}=\frac{2}{3}+\frac{4}{9}\right.$

2 $\left\{\frac{16+15+12+7}{16+16+16+16}=\frac{50}{64}=\frac{2}{3}+\frac{15}{32}.\right.$

3 $\left\{\frac{25+24+21+16+9}{25+25+25+25+25}=\frac{95}{125}=\frac{2}{3}+\frac{7}{27}\right.$

4 $\left\{\frac{36+35+32+27+20+11}{36+36+36+36+36+36}=\frac{161}{216}=\frac{2}{3}+\frac{17}{72}.\right.$

From theſe Examples it plainly appears, that as the Number of Terms in the Series increaſes, the Fraction or Exceſs above $\frac{2}{3}$ decreaſes; and if we ſuppoſe the Series to be infinitely continued, that Exceſs will quite vaniſh, and the Sum of the infinite Series will be $\frac{2}{3}$ of ſo many equal to the greateſt.

CHAP. II.

The Mensuration of SOLIDS.

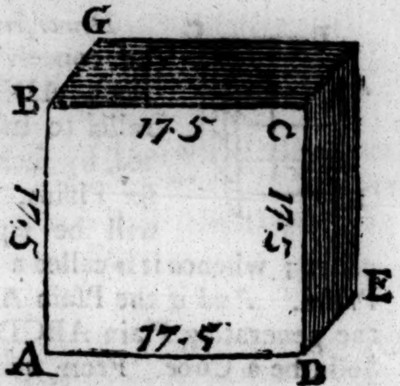
SOLID Bodies are such as do consist of Length, Breadth, and Thickness; as Stone, Timber, Globes, Bullets, *&c.*

§ I. *Of a* CUBE.

A CUBE is a square Solid, comprehended under six geometrical Squares, being in the Form of a Dye. To find the solid Content, this is

The RULE.

Multiply the Side of the Cube into itself, and that Product again by the Side; the last Product will be the Solidity, or solid Content of the Cube.



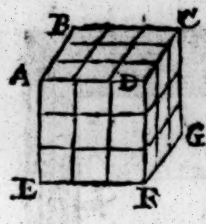
$$
\begin{array}{r}
17.5 \\
17.5 \\
\hline
875 \\
1225 \\
175 \\
\hline
306.25 \\
17.5 \\
\hline
153125 \\
214375 \\
30625 \\
\hline
5359.375
\end{array}
$$

5359·375 the solid Content of the Cube.

Suppose ABCDEFG a cubical Piece of Stone or Wood, each Side thereof being 17 Inches and an half; multiply 17.5 by 17.5, and the Product is 306.25; which being multiply'd by 17.5, the last Product is 5359·375, which is 5359 solid Inches and 375 Parts. To reduce the solid Inches to Feet, divide by 1728, (because so many cubical Inches is a Foot) and the solid Feet in the Cube will be 3, and 175 cubical Inches remain.

By Scale and Compasses.

Extend the Compasses from 1 to 17.5; that Extent turn'd over twice from 17.5 will reach to 5359, the solid Content in Inches. Then extend the Compasses from 1728 to 1; that Extent, turn'd the same way from 5359, will reach to 3.1 Feet.

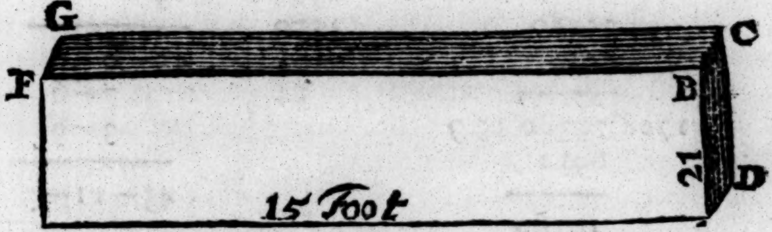


Demonſtration. If the Square ABCD be conceiv'd to be mov'd down the Plain ADEF, always remaining parallel to itſelf, there will be generated, by ſuch a Motion, a Solid, having ſix Plains, the two oppoſite whereof will be equal and parallel to each other; whence it is called a Parallelopipedon, or ſquare Priſm. And if the Plain ADEF be a Square equal to the generating Plain ABCD, then will the generated Solid be a Cube. From hence ſuch Solids may be conceived to be conſtituted of an infinite Series of equal Squares, each equal to the Square ABCD; and AE or DF will be the Number of Terms, Therefore, if the Area of ABCD be multiply'd into the Number of Terms AE, the Product is the Sum of all the Series, (*per* Lemma I.) and, conſequently, the Solidity of the Parallelopipedon or Cube. Or, if the Baſe ABCD, being divided into little ſquare Areas, be multiply'd into the Height AE, divided by a like Meaſure for Length, after this Way you may conceive as many little Cubes to be generated in the whole Solid, as is the Number of the little Areas of the Baſe multiply'd by the Number of Diviſions the Side AE contains. Thus, if the Side of the Baſe AB be 3, that multiply'd into itſelf is 9, which is the Area of the ſquare Baſe ABCD; then, if AE be likewiſe 3, multiply 9 by 3, and the Product is 27; and ſo many little Cubes will this Solid be cut into, if you conceive it to be cut as the Lines direct.

From this Demonſtration it is very plain, that if you multiply the Area of the Baſe of any Parallelopipedon into its Length or Height, that Product will be the ſolid Content of ſuch a Solid.

§ II. *Of a* Parallelopipedon.

LET ABCDEFG be a Parallelopipedon, or ſquare Priſm, repreſenting a ſquare Piece of Timber or Stone, each Side of its ſquare Baſe ABCD being 21 Inches, and its Length AE 15 Foot.



Firſt, then, multiply 21 by 21, the Product is 441, the Area of the Baſe in Inches; which multiply'd by 180, the Length in Inches, and the Product is 79380, the ſolid Content in Inches. Divide the laſt Product by 1728, and the Quotient is 45.9, that is, 45 ſolid Feet and 9 Tenths of a Foot. Or thus: Multiply 441 by 15 Feet, and the Product is 6615; divide this by 144, and the Quotient is 45.9, the ſame as before.

Or thus, by multiplying Feet and Inches.

Multiply 1 Foot 9 Inches by 1 Foot 9 Inches, and the Product is 3 Feet 0 Inches 9 Parts; this multiply'd again by 15 Feet, gives 45 Feet 11 Inches 3 Parts, that is, 45 Feet and $\frac{11}{12}$ of a Foot and $\frac{1}{4}$ of $\frac{1}{12}$.

See the Work of all theſe.

21	441	F. I.
21	15	1—9
21	2205	1—9
42	441	1—9
441	144)6615(45.9	1—3—9
180	855	3—0—9
35280	1350	15
441	54	45—0—0
1728)79380(45.9		7—6
6912		3—9
10260		45—11—3
8640		
16200		
15552		
648		

By Scale and Compaſſes.

Extend the Compaſſes from 12 to 21, and that Extent will reach to near 46 Feet, being twice turn'd over from 15 Feet; ſo the ſolid Content is almoſt 46 Feet.

If the Baſe of the ſquar'd Solid be not an exact Square, but in form of a rectangle Parallelogram, the Way of meaſuring of it is much the ſame; for, firſt, you muſt find the Area of the Baſe by multiplying the Breadth by the Depth; and then multiply that Area by the Length of the Piece, as before: Thus,

If

If a Piece of Timber be 25 Inches broad, 9 Inches deep, and 25 Feet long, how many ſolid Feet are contained therein ?

25	F. I.
9	2—1
	0—9
225	
25	1—6—9
	25
1125	
450	25—0—0
	12—6—0
144)5625(39	1—0—6
432	0—6—3
1305	39—0—9
1296	
09	Anſwer 39 Feet.

By Scale and Compaſſes.

Firſt, find a mean geometrical Proportion between the Breadth and the Depth ; which to do upon the Line of Numbers, you muſt divide the Space upon the Line, between the Breadth and Depth, into two equal Parts ; that middle Point will be the mean Proportional ſought : Thus the middle Point between 25 and 9 is at 15 ; ſo is 15 a mean Proportional between 9 and 25, for, 9 : 15 :: 15 : 25 ; ſo a Piece of Timber of 15 Inches ſquare is equal to a Piece 25 Inches broad and 9 Inches deep. So then, if you extend the Compaſſes from 12 to 15, that Extent, turn'd twice over from 25 Feet, the Length, will reach to to 39 Feet, the Content.

§ III.

§ III. *Of a Triangular* Prism.

A Priſm is a Solid contained under ſeveral Plains, and having its Baſes like, equal, and parallel. The ſolid Content of a Priſm (whether triangular or multangular) is found by multiplying the Area of the Baſe into the Length or Height, and the Product is the ſolid Content.

Let ABCDEF be a triangular Priſm, each Side of the Baſe being 15.6 Inches, the Perpendicular thereof C *a* is 13.51 Inches, and the Length of the Solid 19.5 Feet.

Multiply the Perpendicular of the Triangle 13.51 by half the Side 7.8, and the Product is 105.378, the Area of the Baſe; which multiply by the Length 19.5, and the Product is 2054.871, which divide by 144, and the Quotient is 14.27 Feet *fere*, the ſolid Content.

13.51	144)2054.87(14.27
7.8	144
10808	614
9457	576
105.387	388
19.5	288
526890	1007
948402	1008
105378	
2054.8710	

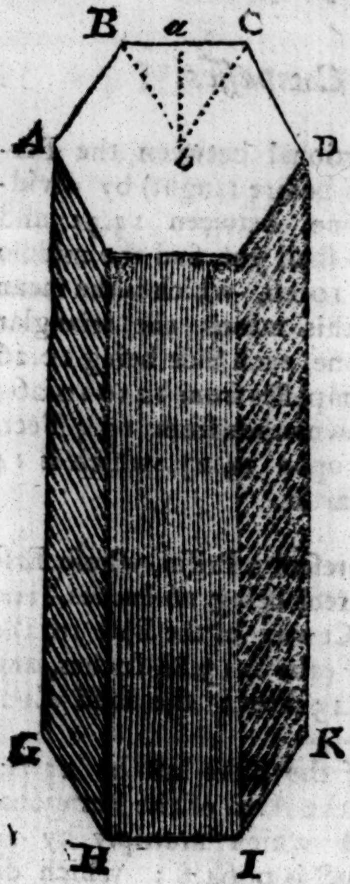
By

By Scale and Compaſſes.

Firſt, find a mean Proportional between the Perpendicular and Half-ſide, (as before taught) by dividing the Space upon the Line, between 13.51 and 7.8 into two equal Parts; ſo ſhall you find the middle Point between them to be at 10 26, which is the mean Proportional ſought: By this means the trianglar Solid is brought to a ſquare one, each Side being 10.26 Inches. Then extend the Compaſſes from 12 to 10.26; that Extent, turn'd twice downwards from 19.5 Feet, the Length will at laſt fall upon 14.27, which is 14 Feet and a little above a Qnarter.

Let ABCDEFGHIK repreſent a Priſm, whoſe Baſe is a Hexagon, each Side thereof being 16 Inches, and the Perpendicular from the Centre of the Baſe to the Middle of one of the Sides, (ab) is 13.84 Inches, and the Length of the Priſm is 15 Feet; the ſolid Content is required.

Multiply half the Sum of the Sides 48 by 13.84, and the Product is 664.32, the Area of the hexagonal Baſe, (by § VIII. *p.* 86.) which multiply by 15 Feet, the Length, the Product is 9964.8; which divided by 144, the Quotient will be 69.2 Feet, the ſolid Content required.



13.84
48

11072
5536

664.32 Area of Bafe.
15

332160
66432

144)9964.80(69.2
864

1324
1296

288
288

By Scale and Compaffes.

Firft, find a mean Proportional between the Perpendicular, and half the Sum of the Sides; that is, divide the Space between 13.84 and 48, and the middle Point will be 25.77. Then extend the Compaffes from 12 to 25.77; that Extent will reach (being twice turn'd over) from 15 Feet, the Length, to 69.2 Feet, the Content.

To

To find the ſuperficial Content of any of the forementioned Solids, you muſt take the Girth of the Piece, and multiply by the Length, and to that Product add the two Areas of the Baſes, the Sum will be the whole ſuperficial Content. Example of the hexagonal Priſm laſt-mentioned: The Sum of the Sides being 96, and the Length 15 Feet, that is, 180 Inches; which multiply'd by 96, the Product is 17280 ſquare Inches; to which add twice 664.32, the Areas of the two Baſes, and the Sum is 18608.64, the Area of the Whole, which is 129.22 Feet.

```
        180
         96
      -----
       1080
      1620
      -----
      17280
        664.32
        664.32
      --------
144)18608.64(129.22
      ----
      420
      1328
        326
         384
        ----
          96
```

The ſuperficial Content of the whole Solid is 129.22 Feet.

By Scale and Compaſſes.

Extend the Compaſſes from 144 to 180; that Extent will reach from 96 to 120 Feet. Then, to find the Area of the Baſe, extend the Compaſſes from 144

 to

to 13.84; that Extent will reach from 48 to 4.6 Feet; add 120 Feet, and twice 4.6 Feet, and it makes 129.2 Feet, the ſuperficial Content, as before.

The Demonſtration of thoſe laſt Solids will be the ſame as in the firſt Section; for as in that, ſo in theſe, the Area of the Baſe is multiply'd into the Length to find the Content, and the ſame Reaſon is given for one as for the other.

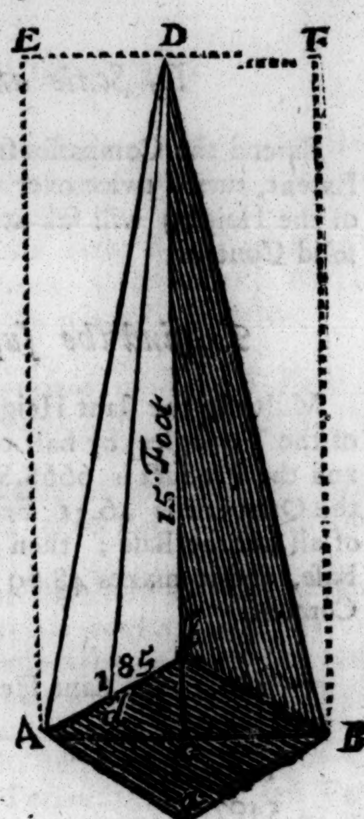
§ IV. *Of a* PYRAMID.

A Pyramid is a ſolid Figure, whoſe Baſe is a Polygon, and whoſe Sides are plain Triangles, their ſeveral Tops meeting together in one Point. To find the ſolid Content thereof, this is

The RULE.

Multiply the Area of the Baſe by a third Part of the Altitude, or Length; and the Product is the ſolid Content of the Pyramid.

Let



Let ABD be a ſquare Pyramid, each Side of the Baſe being 18.5 Inches, and the perpendicular Height CD is 15 Feet: Multiply 18.5 by 18.5, and the Product is 342.25, the Area of the Baſe in Inches; which multiply'd by 5, a third Part of the Height, and the Product is 1711.25; this divided by 144, the Quotient is 11.88 Feet, the ſolid Content.

18.5
18.5
———
925
1480
185
———
342.25 Area of the Baſe.
5
———
144)1711.25(11.88 Content.

F.	I.	Pts.	
1	6	6	
1	6	6	
1	6	6	
	9	3	
		9	3
2	4	6	3
			5
11	10	7	3

By Scale and Compaſſes.

Extend the Compaſſes from 12 to 18.5 Inches, that Extent, turn'd twice over from 5 Feet, (a third Part of the Height) will fall at laſt upon 11.88 Feet, the ſolid Content.

To find the ſuperficial Content.

Multiply the ſlant Height (or Perpendicular of one of the Triangles) by half the Periphery of the Baſe 37, and the Product is 6668.88, which divided by 144, the Quotient is 46.31 Feet, the ſuperficial Content of all but the Baſe; then to that add 2.38 Feet the Baſe, and it makes 48.69 Feet, the whole ſuperficial Content.

```
   180.24 the ſlant Height d D
       37                              144)342.25(2.38
  -------                                  288
   126168                                  ---
   54072                                   542
  -------                                  432
144)6668.88(46.31                          ----
   576        2.38                         1105
   ---        ----                         1152
   908       48.69 the whole Content.
   864
   ---
    448
    432
    ---
     168
     144
     ---
      24
```

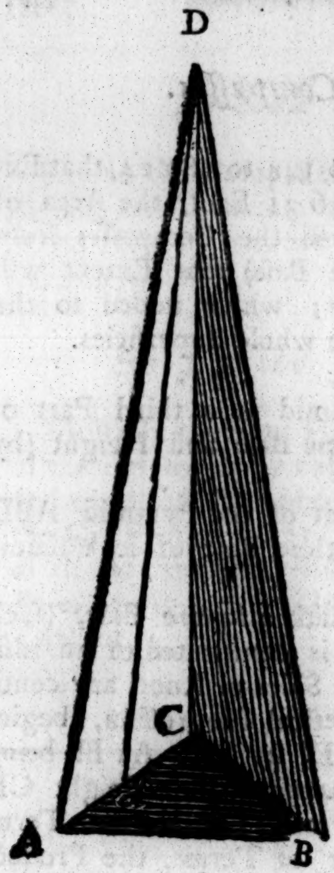
By Scale and Compaſſes.

Extend the Compaſſes from 144 to 180.24, that Extent will reach from 37 to 46.31 Feet, the Area of the four Triangles; and extend the Compaſſes from 144 to 18.5 (one Side of the Baſe) that Extent will reach from 18.5 to 2.38 *fere*; which added to the other, the Sum is 48.69, the whole Superficies.

Demonſtration. Every Pyramid is a third Part of the Priſm, that hath the ſame Baſe and Height (by *Eucl.* 12.7.)

That is, the ſolid Content of the Pyramid ABD (in the laſt Figure) is one third Part of its circumſcribing Priſm ABEF.

For every Pyramid that hath a ſquare Baſe, (ſuch as Aa Bb in the laſt Figure) is conſtituted of an infinite Series of Squares, whoſe Sides or Roots are continually increaſing in arithmetical Progreſſion, beginning at the Vertex or Point D, its Baſe Aa Bb being the greateſt Term, and its perpendicular Height CD is the Number of all the Terms: But the laſt Term multiply'd into the Number of Terms, the Product will be triple the Sum of all the Series (by Lemma 3.); conſequently $\frac{NLL}{3} = S$. And S is equal to the ſolid Content of the Pyramid. From hence it will be eaſy to conceive, that every Pyramid is $\frac{1}{3}$ of its circumſcribing Priſm, (that is, of a Priſm of equal Baſe and Altitude) what Form ſoever its Baſe is of, *viz.* whether it be ſquare, triangular, pentangular, *&c.* You may very eaſily prove a triangular Pyramid to be a third Part of a Priſm of equal Baſe and Altitude, by cutting a triangular Priſm of Cork, and then cut that Priſm into three Pyramids, by cutting diagonally, as I have ſeveral times done, to ſatisfy myſelf and others.



Let ABCD be a triangular Pyramid, each Side of the Baſe being 21.5 Inches, and its Perpendicular Height 16 Feet; the Content, ſolid and ſuperficial, is required.

Firſt, find the Area of the Baſe, by multiplying half the Side by the Perpendicular let fall from the Angle of the Baſe to the oppoſite Side; which Perpendicular will be found to be 18.62; the Half thereof is 9.31, multiply'd by 21.5, the Product is 200.165 Inches, the Area of the Baſe. Then, becauſe the Altitude 16 cannot exactly be divided by 3, therefore I take the third Part of 200.165, which is 66.72, and multiply it by 16, and the Product is 1067.52, whieh divided by 144, the Quotient is 7.41 Feet, the ſolid Content.

```
9.31 Half the Perpend.
21.5 the Side.
————
4655
 931
1862
————
3)200.165 Area Base.
————
66.72 a third Part.
   16 Height.
————
40032
6672
————
144)1067.52(7.41 solid Cont.
1008
————
 595
 576
————
 192
 144
————
  48
```

	F.	I.	Pts.	
Side	1	9	6	
Half Perp.		9	4	
	1	4	1	6
			7	2
Area Base	1	4	8	8
				4
	5	6	10	8
				4
	3)22	3	6	8
Content	7	5	2	2

In casting this up by Feet and Inches, instead of multiplying by 16 the Height, I break 16 into two such Numbers, as, being multiplied together, the Product may be 16, *viz.* into 4 and 4, and multiply first by one, and then the other; a third Part of the last Product is the Content.

By Scale and Compasses.

First, find a geometrical mean Proportional, (as before directed) by dividing the Space between 21.5 and 9.31 into two equal Parts, and you will find the middle Point at 14.15, which is the mean Proportional sought. Then extend the Compasses from 12 to

14.15, that Extent (turn'd twice over from 16 Feet) will fall at laſt upon 22,23; a third Part thereof is 7.41 Feet, the Content.

To find the ſuperficial Content.

Multiply the ſlant Height (or Perpendicular of one of the Triangles) by half the Periphery of the Baſe, and to that Product add the Area of the Baſe, the Sum is the whole ſuperficial Content.

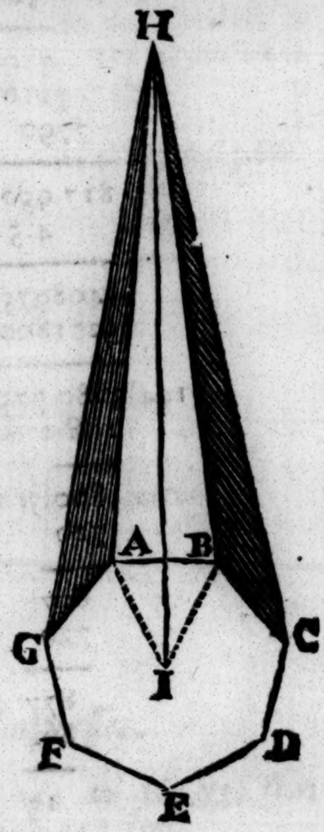
```
              192.1 Inches, the ſlant Height d D.
Half Periph.  32.25=21.5+10.75.
              ------
               9605
              3842
             3842
            5763
            -----------
            6195.225 Inches, the Area of all but the Baſe.
             200.165 Area of the Baſe add.
            --------
        144)6395.390(44.41 Feet, the whole Content.
            576
            ---
             635
             576
             ---
              593
              576
              ---
               179
               144
               ---
                35
```

By Scale and Compaſſes.

Extend the Compaſſes from 144 to 192.1, that Extent will reach from 32.25 (half the Periphery of the Baſe) to 43.02 Feet, the Content of the upper Part.

 And

And extend the Compasses from 144 to half the Perpendicular 9.31, that Extent will reach from the Side 21.5 to 1.39 Feet, the Area of the Base; which, added to the other, makes 44.41 Feet, the Content of the Whole.



Let ABCDEFGH be a Pyramid, whose Base is a Heptagon, each Side thereof being 15 Inches, and the Perpeudicular of the Heptagon is 15.58 Inchəs, and the perpendicular Height of the Pyramid HI is 13.5 Feet; the Content solsd and superficial is required.

Multiply 15.58 (the Perpendicular) by 52.5, (half the Sum of the Sides of the Heptagon) and the Product is 817.95, which multiply'd by 4.5, *viz.* $\frac{1}{3}$ of the Height, and the Prodnct is 3680.775.

Then

Then divide this laft Product by 144, and the Quotient is 25.56 Feet, the Content.

```
         15.58 the Heptagon's Perpendicular.
          52.5 the Half Sum of the Sides.
        ------
          7790
         3116
        7790
        ------
       817.950
           4.5 a third Part of the Height.
       --------
       4089750
      3271800
      --------
144)3680.7750(25.56 folid Feet.
    288
    ---
     800
     720
     ---
      807
      720
      ---
       877
       864
       ---
        13
```

By Scale and Compaffes.

Firft, find a geometrical mean Proportional between 15.58 and 52.5, (as is before directed) which you will find to be 28.06; then extend the Compaffes from 12 to 28.06, that Extent will reach from 4.5 (twice turn'd over) to 25.56 Feet.

To

To find the ſuperficial Content.

Multiply the Height taken from the Middle of one of the Sides of the Baſe 162.75 Inches, by the Half-Sum of the Sides 32.5 Inches, and the Product is 8544.375; which divided by 144, the Quotient is 59.335 Feet, the Content of the upper Part.

```
      162.75                          144)817.95(5.68
       52.5                                979
      ------                               1155
       81375                               ----
      32550                                  3
     81375
     -------
144)8544.375(59.335 Feet.
    ----        5.68 Baſe add.
     1344      ------
      483      65.015 the whole Content.
       517
        855
        ---
        135
```

By Scale and Compaſſes.

Extend the Compaſſes from 144 to 162.75, that Extent will reach from 52.5 to 59.335 Feet.

And extend the Compaſſes from 144 to 15.58, the Perpendicular of the Heptagon, that Extent will reach from 5.25 to 5.68 Feet, the Content of the Baſe; which add to the former, the Sum is 65.015, the whole ſuperficial Content.

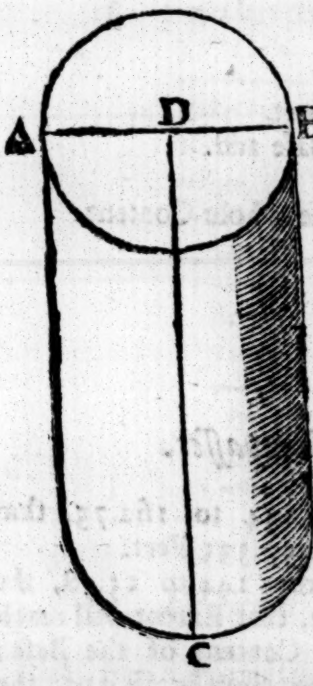
§. V.

§ V. *Of a* Cylinder.

A Cylinder is a round Solid, having its Baſes circular, equal and parallel, in form of a Rolling-ſtone uſed in Gardens. To find the ſolid Content thereof, this is

The RULE.

Multiply the Area of the Baſe by the Length, and the Product is the ſolid Content.



Let A B C be a Cylinder, whoſe Diameter A B is 21.5 Inches, and the Length C D is 16 Feet; the ſolid Content is required.

Firſt, ſquare the Diameter 21.5, and it makes 462.25; which multiply by .7854, and the Product is 363.05115. Then multiply this by 16, and the Product is 5808.8164. Divide this laſt Product by 144, and the Quotient is 40.34 Feet, the ſolid Content.

By Scale and Compaſſes.

Extend the Compaſſes from 13.54 to 21.5, the Diameter, that Extent (turn'd twice over from 16, the Length) will at laſt fall upon 40.34, the ſolid Content.

To

To find the ſuperficial Content.

Firſt, (by Chap. I. Sect. IX. Prob. 2.) find the Circumference of the Baſe 67.54, which multiply by 16. the Product is 1080.64; which divided by 12, the Quotient is 90.05 Feet, the curve Surface; to which add 5.04 Feet, the Sum of the two Baſes, and the Sum is 95.09 Feet, the whole ſuperficial Content.

67.54		363.05
16		2
40524		144)726.10(5.04
6754		
	90.05 } add	610
12)1080.64	5.04	
90.05	95.09	34

By Scale and Compaſſes.

Extend the Compaſſes from 12 to 67.54, (the Circumference) that Extent will reach from 16 (the Length) to 91.05 Feet, the curve Surface.

And extend the Compaſſes from 12 to 21.5, (the Diameter) that Extent (turn'd twice from .7854) will at laſt fall upon 2.52 Feet, the Area of one Baſe; which doubled is 5.04; this, added to the curve Surface, makes 95.09 Feet, the whole ſuperficial Content.

Demonſtration, The ſolid Content of every Cylinder is found, by multiplying the Area of its Baſe into its Height, as aforeſaid: For every right Cylinder is only a round Priſm, being conſtituted of an infinite Series of equal Circles; that of its Baſe, or End, being one of the Terms, and its Height CD (in the former Figure) is the Number of all the Terms. Therefore the Area of its Baſe AB being multiply'd into CD,

CD, will be its Solidity (by Lemma I.) Let D=AB, and H=CD

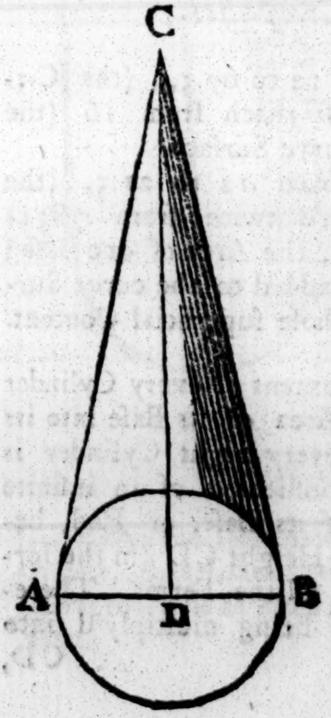
Then .7854 DD×H=its Solidity.

§ VI. *Of a* Cone.

A Cone is a Solid, having a circular Baſe, and growing ſmaller and ſmaller, till it ends in a Point which is called the Vertex, and may be nearly repreſented by a Sugar-loaf. To find the Solidity thereof, this is

The RULE.

Multiply the Area of the Baſe by a third Part of the perpendicular Height, and the Product is the ſolid Content.



Let ABC be a Cone, the Diameter of whoſe Baſe AB is 26.5 Inches, and the Height of the Cone DC is 16.5 Feet: Firſt, ſquare the Diameter 26.5, and it is 702.25, which multiply by .7854, and the Product is 551.547715; which multiply by 5.5, and the Product is 3033.478525; which divided by 144, the Quotient is 21-07 *fere*, the ſolid Content of the Cone.

2.65

26.5 the Diameter.
26.5

1325
1590
530

702.25 the Square.
.7854

280900
351125
561800
491575

551.54|715 Area of the Base.
5.5 a third Part of the Height.

275773
275773

144)3033.503(21.07 Feet, the Content.

153
947

By Scale and Compasses.

Extend the Compasses from 13.54 to 26.5, the Diameter) that Extent, turn'd twice over from 5.5, (a third Part of the Height) will at last fall upon 21.07 Feet, the Content.

To find the superficial Content.

Multiply half the Circumference 41.626 by the slant Height A C 198.46, and the Product is 8261.09596; which, divided by 144, the Quotient is 57.37 *fere*, the curve Surface; to which add the Base, the Sum is 61.2, the superficial Content.

```
       41.626 the Half-circumference of the Base.
       198.46 the slant Height.
       ------
       249756
      166504
     333008
    374634
    41626
    -----------
144)8261.09596(57.37 Feet fere.
    ----          3.83 the Base add.
    1061         -----
      530        61.20 the whole Content.
       989
     ------
144)561.54(3.83
     -----
     1195
       434
       ---
         2
```

By Scale and Compasses.

Extend the Compasses from 144 to 198.46, that Extent will reach from 41.626 to 57.37 Feet, the curve Surface.

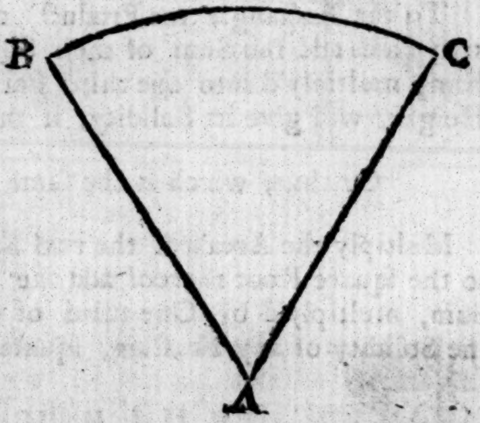
And extend the Compasses from 12 to 26.5 the Diameter; that Extent, turn'd twice over from .7854, will at last fall upon 3.83 Feet, the Base; which added to 57.37, the Sum is 61.2 Feet, the superficial Content.

Demonstration. Every Cone is the third Part of a Cylinder of equal Base and Altitude. The Truth of this may easily be conceiv'd, by only considering, that a Cone is but a round Pyramid; and therefore it must needs have the same Ratio to its circumscribing Cylinder, as the square Pyramid hath to its circumscribing

ſcribing Parallelopipedon, *viz.* as 1 to 3. However, to make it yet clearer, let it be farther conſider'd, That

Every right Cone is conſtituted of an infinite Series of Circles, whoſe Diameters do continually increaſe in Arithmetical Progreſſion, beginning at the Vertex, or Point C, the Area of its Baſe A B being the greateſt Term, and its perpendicular Height D C, the Number of all the Terms; therefore the Area of the Circle of the Baſe, multiply'd by a third Part of the Altitude DC, will be the Sum of all the Series, equal to the Solidity of the Cone, by Lemma III.

The curve Superficies of every right Cone is eqal to half the Rectangle of the Circumference of its Baſe into the Length of its Side.



For the curve Surface of every right Cone is equal to the Sector of a Circle whoſe Arch BC is equal to the Periphery of the Baſe of the Cone, and Radius A B equal to the ſlant Side of the Cone. Which will appear very evident, if you cut a Piece of Paper in the Form of a Sector of a Circle, as ABC, and bend both the Sides AB and AC together, till they meet, and you will find it to form a right Cone.

I have omitted the Demonſtrations touching the Superficies of all the foregoing Solids, becauſe I thought it needleſs, they being all compos'd of Squares, Parallelograms, Triangles, &c. which Figures are all demonſtrated before. And if the Area of all ſuch Fi-

P 2 gures

gures as compoſe the Solid, be found ſeverally, and added together, the Sum will be the ſuperficial Content of the Solid.

§ VII. *Of the Fruſtum of a* PYRAMID.

A Fruſtum of a Pyramid is the remaining Part, when the Top is cut off by a Plain parallel to the Baſe. To find the Solid Content thereof, there are ſeveral Rules.

RULE I.

To the Rectangle (or Product) of the Sides of the two Baſes add the Sum of their Squares; that Sum, being multiply'd into one third Part of the Fruſtum's Height, will give its Solidity, if the Baſes be ſquare.

Or thus, which is the ſame in Effect:

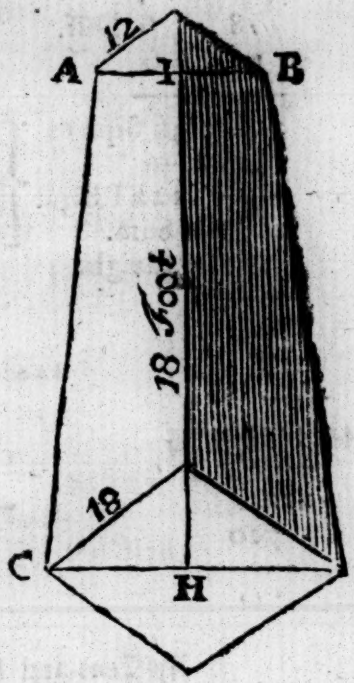
Multiply the Area's of the two Baſes together, and to the ſquare Root rhereof add the two Areas; that Sum, multiply'd by One-third of the Height, gives the Solidity of any Fruſtum, ſquare or multangled.

RULE II.

To the Rectangles of the Sides of the two Baſes, add one-third Part of the Square of their Difference; that Sum being multiply'd into the Height, will produce the Solidity, if the Baſes be Squares: But if they be triangular or multangular, the ſaid Rectangle of the Sides, with the third Part of the Square of their Difference, will be the Square of a mean Side; and the ſquare Root thereof will be ſuch a mean Side as

as will reduce the tapering Solid to a Priſm equal thereunto.

Example. Let ABCD be the Fruſtum of a ſquare Pyramid, the Side of the greater Baſe 18 Inches, and the Side of the leſſer 12 Inches, and the Height 18 Feet; the Solidity thereof is required.



Firſt, multiply the two Sides together, 18 by 12, and the Product is 216, and the Difference of the Sides is 6, whoſe Square is 36; a third Part thereof is 12, which added to 216, the Sum is 228 Inches, the Area of a mean Baſe; which multiply'd by 18 Feet the Length, the Product is 4104; this divided by 144, the Quotient is 28.5 Feet the Content.

Or, by the firſt Rule, thus; the Square of 18 is 324, and the Square of 12 is 144, and the Rectangle of 18 by 12 is 216; the Sum of theſe three is 684, which multiply'd by 6, the Product is 4104; which divided by 144, the Quotient is 28.5 Feet, the ſame as before.

See the Work of both Ways.

18	6 Diff.		18	12
12	6		18	12
216	3)36 Square		324 Sq.	144 Sq.
12 add	12 a Third.		144	
228 the Sum.			216	
18 the Height.			684 the Sum.	
1824			6 a 3d of the Height.	
228			144)4104(28.5 Feet.	
144)4104(28.5			1224	
1224			720	
720			...	
...				

By Feet and Inches thus:

	F.	I.	I.
Mult.	1	6	6
by	1		6
Prod.	1	6	3)36 q.
add	0	1	12
Mult.	1	7	
by	18	0 Height.	
	18	0	
	9	0	
	1	6	
Cont.	28	6	

Or thus;

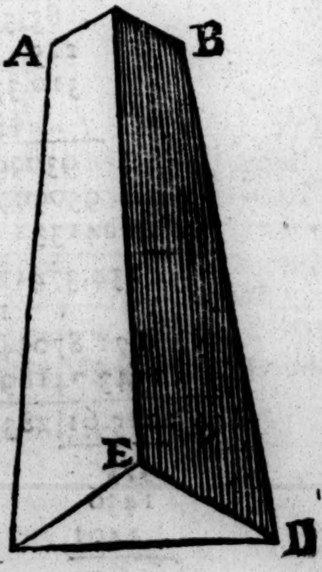
F.	I.	
2	3	Sq. of the greater.
1	6	the Rectangle.
1	0	Sq. of the leſs.
4	9	Trip. of a mean Ar.
6	0	a 3d of the Height.
28	6	

To find the ſuperficial Content.

The Perimeter of the greater Baſe is 72, and the Perimeter of the leſſer Baſe is 48; add both the Perimeters together, the Sum is 120; the Half thereof is 60; which multiply'd by 18 Feet, the Product is 1080; this divided by 12, the Quotient is 90 Feet; to which add the two Baſes 2.25 Feet, and 1 Foot, the Sum is 93,25 Feet, the whole ſuperficial Content.

18	12		18 the Height.
4	4		60
72	48	12)	1080
48			90 Feet.
			2.45 the greater Baſe.
2) 120			1 the leſſer Baſe.
60			93.25 the Sum.

Again; Let ABC be the Fruſtum of a triangular Pyramid, each Side of the greater Baſe 25 Inches, and each Side of the leſſer Baſe 9 Inches, and the Length 15 Feet; the ſolid Content thereof is required.



By the ſecond Rule, multiply 25 by 9, and the Product is 225; and the Difference between 25 and 9 is 16; which, ſquar'd, makes 256, a third Part thereof is 85.333, which added to 225, the Sum is 310.333; and this multiply'd by 433, the

Product

Product is 134·374, *&c.* which is the Area of a mean Base; and that multiply'd by 15 Feet, the Length, the Product is 2015.610; which divided by 144, the Quotient is 13.99 Feet, the Solidity.

Or thus, by the latter Part of the first Rule: Find the Area of the greater Base, which you will find to be 270.625, and the Area of the lesser Base will be 35·073; these two Area's multiply'd together, the Product is 9491.630625; the square Root thereof is 97·425; to which add the two Area's, and the Sum is 403.123; which multiply'd by a third Part of the Length 5, the Product is 2015.615; and that divided by 144, the Quotient is 13.99 Feet, as before.

See the Working of both.

```
              25            25
               9             9
Product      225            16  Diff.
                            16
                            96
                           16
                        3)256  the Square.
                       85·333  a third Part.
                      225      add.
                      310·333
                         ·433  Tabular Number (vid. p. 89)
                       930999
                      930999
                    1241332
                   134·374189  mean Area.
                           15  Length.
                  671.870945
                 1343·74189
         144)2015.61|2835(13.99 Feet.
             575
             1436
              1401
              105
```

25
25
———
125
50
———
625 Square.
433
———
1875
1875
2500
———
270.625 Area.

9
9
—
81 Square.

433
81
———
433
3464
———
35.073

270.625
35.073
———
811875
1894375
13531250
811875
———
9491.630625

9491.630625(97.425
81
———
187) 1391
1309
———
1944) 8263
7776
———
19482)48706
38964
———
194845)974225
974225
———
......

270.625 greater Area.
97.425 the mean Proportional.
35.073 the leſſer Area.

403.123 the Triple of a mean Area.
5 a third Part of the Height.

144)2015.615(13.99 Feet, the Solidity.
575
1436
1431

105

In finding the Area of the triangular Baſe, I multiply by .433, becauſe that is the Area of the equilateral Triangle, when the Side thereof is 1. A Table of the Area's, or Multipliers, for finding the Area's of Polygons, you'll find in *p.* 89.

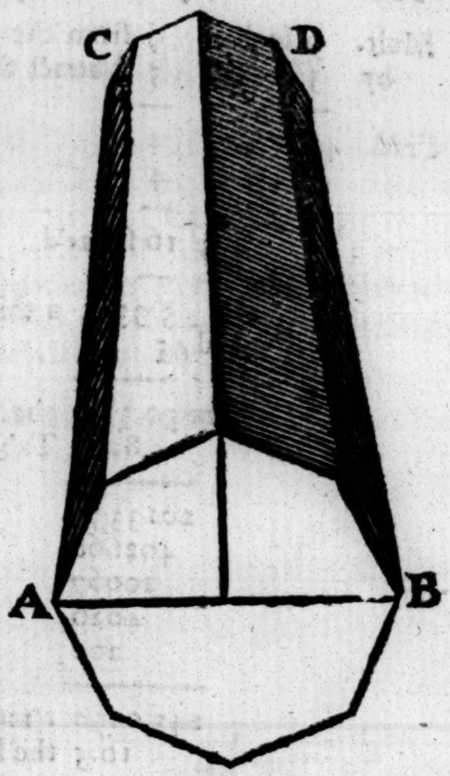
Multiply the Square of the Side by the tabular Number, and the Product is the Area of the Polygon.

To find the ſuperficial Content.

The Perimeter of the greater Baſe is 75, and the Perimeter of the leſſer Baſe is 27; the Sum of both is 102, and the Half-ſum is 51; which multiply'd by 15 Feet, the Product is 765; which divided by 12, the Quotient is 63.75; to which add the Sum of the two Baſes 2.12 Feet, and the Sum is 65.87 Feet, the whole ſuperficial Content.

Note, That 51 ſhould have been multiply'd by the ſlant Height, but the Difference it would make, is but .06 of a Foot, which is inconſiderable.

Again:



Again: Suppoſe ABCD to be the Fruſtum of a Pyramid, having an octagonal Baſe, each Side thereof being 9 Inches, and each Side of the leſſer Baſe 5 Inches, and the Length or Height 10.5 Feet, the Solidity is requir'd.

By the ſecond Rule, multiply the greater Side 9 by the leſſer Side 5, and the Product is 45; then the Difference between 9 and 5 is 4, which ſquar'd makes 16; a third Part thereof is 5.3333, which added to 45, the Sum is 50.3333; multiply this laſt by the Number in the Table 4.8284, and the Product is 243.0292, the Area of a mean Baſe; which multiply'd by the Height 10.5 Feet, the Product is 2551.8066; then divide this laſt Product by 144, and the Quotient is 17.72 Feet, the ſolid Content.

See the Work.

Mult.

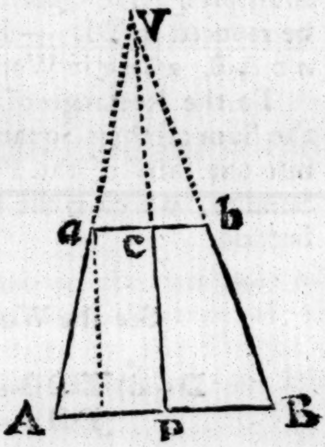
Mult.	9	Inches.	9	from the greater Side.
by	5	Inches.	5	subtract the lesser.
Prod.	45		4	

```
                4
                4
              3)16 squar'd.
             5.3333 a third Part.
         Add 45
         Sum 50.3333 the Square of a mean Side.
              4.8284 Tabular Number, p. 89.
           2013332
            402666
             10067
              4026
               201
          243.0292 a mean Area.
              10.5 the Height.
          12151460
         2430292
     144)2551.80[660(17.72
         144
         1111
         1008
          1038
          1008
            300
            288
             12
```

To find the ſuperficial Content.

The Perimeter of the greater Baſe is 72, and the Perimeter of the leſſer Baſe is 40, and their Sum is 112; the Half thereof is 56, which multiply'd by the Height 10.5 Feet, and the Product is 588; which divided by 12, the Quotient is 49 Feet; to which add the Sum of the two Baſes, 3.55, and the Sum is 52.55 Feet, the whole ſuperficial Content.

Demonſtration. From the Rules delivered in the IVth and VIth Sections, the two foregoing Rules may eaſily be demonſtrated.



Suppoſe a ſquare Pyramid, ABV, to be cut by a Plain at *a* b, parallel to its Baſe AB, and it were requir'd to find the Solidity of the Fruſtum, or Part *a* b AB. Let there be given

D=BA, the Side of the greater Baſe.
d=b*a*, the Side of the leſſer Baſe.
H=CP, the perpendicular Height.

Firſt,	1	$D-d : H :: d : \frac{dH}{D-d}$ = VC by the Figure.
Then	2	$DD \times \frac{H+VC}{3}$ = the whole Pyram. BVA, by Section the IVth.
And	3	$dd \times \frac{1}{3}VC$ = the Pyramid *a* V b cut off.

Then in the 2d and 3d Steps, if, inſtead of VC, you take $\frac{dH}{D-d}$ equal to it by the firſt Step, it will be,

viz. 1. 2.	4	$\frac{DDDH}{3D-3d}$ = the whole Pyramid BVA.
and 1. 3.	5	$\frac{dddH}{3D-3d}$ = the Pyramid *a* V b.
4.—5.	6	$\frac{DDDH-dddH}{3D-3d}$ = the Fruſtum *a*bAB

And by dividing DDD—ddd by D—d, and then multiplying the Quotient by $\frac{1}{3}$ H, the laſt Step will be reduced to $DD+Dd+dd:\times\frac{1}{3}H$ = the Fruſtum *a* b A B, which in Words is thus ;

To the Rectangle of the Sides of the two Baſes add the Sum of their Squares ; that Sum being multiply'd into one third of the Fruſtum's Height, will give its Solidity, which is the ſame as the firſt Rule of this Section.

See the Work of the Diviſion.

$$
\begin{array}{rl}
D-d) & DDD-ddd\ (DD+Dd+dd \\
 & DDD-DDd \\
\hline
 & \quad DDd-ddd \\
 & \quad DDd-Ddd \\
\hline
 & \qquad Ddd-ddd \\
 & \qquad Ddd-ddd \\
\hline
 & \qquad 0 \quad 0
\end{array}
$$

The

The ſame Reaſon will hold good for all Fruſtums of Pyramids or Cones, whether the Baſe be triangular or multangular, becauſe the Squares of the Sides of any Figure, or the Squares of the Diameters of Circles are proportional to the Area, which proves the latter Part of the ſaid firſt Rule.

Again, to prove the ſecond Rule.

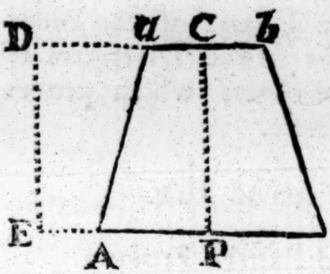
Suppoſe	1	$x = D - d$. And $F =$ the Fruſtum.
then	2	$DD + Dd + dd = \frac{3F}{H}$ by the laſt.
$1 \odot 2$	3	$xx = DD - 2Dd + dd$.
$2 - 3$	4	$3Dd = \frac{3F}{H} - xx$.
$4 \div 3$	5	$Dd = \frac{F}{H} - \frac{1}{3}xx$. Or $Dd + \frac{1}{3}xx = \frac{F}{H}$.
$5 \times H$	6	$\overline{Dd + \frac{1}{3}xx} \times H = F$, the Fruſtum *a*bAb

Which in Words is thus:

To the Rectangle of the Sides of the two Baſes add one third Pert of the Square of the Difference of the ſaid Sides, and multiply the Sum by the Height of the Fruſtum, the Product is the Solsdity of the Fruſtum.

The ſuperficial Contents of Fruſtums, (all but the Baſes) are compos'd of Trapeziums, ſo many as the Fruſtum has Sides. As the ſquare Fruſtum *a*bAb, in the laſt Figure, is compos'd of four Trapeziums, having the two upper, and alſo the two longer Angles equal; if therefore the Trapezium *a*bAB be cut in two by the Line CP, and the two Pieces laid together, the Line bB upon the Line *a*A, the narrow End of the one to the broad End of the other, it will form a right-angled Parallelogram, as is plain by the Figure an-

Q 2 nex'd;



nex'd; the Parallelogram DCEP being equal to the Trapezium *ab*AB; becauſe the Side D*a* is equal to PB, and EA is equal to *a* C. Therefore, to find the Area of the Trapezium, add half the Side *ab* to half the Side AB, and it makes DC or EP; which multiply by the Height PC, the Product is the Area of the Parallelogram DCEP, equal to the Trapezium *ab*AB; then if that be multiply'd by the Number of Trapeziums, the Product will be the ſuperficial Content of the Fruſtum, wanting the Baſes. Or, if the whole Perimeter of the greater Baſe be added to the Perimeter of the leſſer Baſe, and half the Sum multiply'd by Height, the Product will be the ſuperficial Content of all the Trapeziums at once.

Note, That half the Sum of the Perimeters ſhould be multiply'd by the ſlant Height, up the Middle of one of the Trapeziums; but in the foregoing Examples I have multiply'd by the perpendicular Height, becauſe the Difference is very inconſiderable.

§ VIII. *Of the Fruſtum of a* CONE.

A Fruſtum of a Cone, is that Part which remains, when the Top end is cut off by a Plain parellel of the Baſe. To find the ſolid Content, the Rules are the ſame in Effect as for the Fruſtum of a Pyramid.

RULE I.

To the Rectangle of the Diameters of the two Baſes add the Squares of the ſaid Diameters, and multiply the

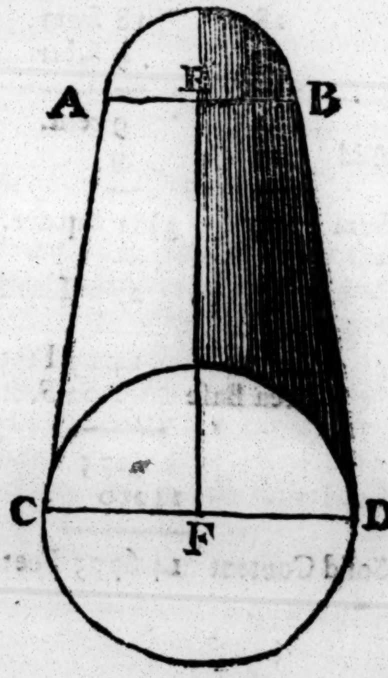
the Sum by .7854, the Product will be the Triple of a mean Area; which multiply'd by $\frac{1}{3}$ of the perpendicular Height, that Product will be the ſolid Content.

Or thus: Multiply the Area's of the greater and leſſer Baſes together, and out of the Product extract the Square Root, and add the two Area's and Square Root together, and multiply the Sum by One-third of the Perpendicular Height, the Product is the ſolid Content.

RULE II.

To the Rectangle of the greater and leſſer Diameters, add one-third Part of the Square of their Difference, and multiply the Sum by .7854, the Product is a mean Area; which multiply'd by the perpendicular Height, the Product is the Solidity.

Example. Let ABCD be the Fruſtum of a Cone, whoſe greater Diameter CD is 18 Inches, and the leſſer Diameter AB 9 Inches, and the Length 14.25 Feet; the ſolid Content is requir'd.



Multiply 18 by 9, and the Product is 162, and the Difference between 18 and 9 is 9, whoſe Square is 81, a third Part is 27; which add to 162, the Sum is 189; this multiply'd by .7854, the Product is 148.44; which divided by 144, the Quotient is 1.03

Feet, the Area of a mean Baſe; which multiply by 14.25 Feet, the Height, the Product is 14.6775 Feet, the ſolid Content.

Or thus, by the firſt Rule.

The Square of 18 (the greater Diameter) is 324, and the Square of 9 (the leſſer Diameter) is 81, and the Rectangle, or Product of 18 by 9, is 162; the Sum of theſe three is 567, which multiply'd by .7854, the Product is 445.3218; which divided by 144, the Quotient is 3.09 Feet, the triple Area of a mean Baſe; this, multiply'd by 4.75 Feet, (a third Part of the Height) and the Product is 14.6775 Feet, the Solidity, the ſame as before.

See the Work.

	18	18 from	.7154
	9	9 ſubtr.	189
	162	9 rem.	70686
Add	27	9	62832
			7854
Sum	189	3)81 Square.	
			144)148.4406(1.03
		27 a Third	144
	Height	14.25 Feet.	444
	Area Baſe	1.03 Feet.	432
		4275	12
		14250	
Solid Content		14.6775 Feet.	

324

324 the Square of 18.
162 the Rectangle.
81 the Square of 9.

567 the triple Square of a mean Diameter.

```
         .7854
           567
        ------
         54978
        47124
       39270
       -------
144)445.32((18      (3.09
    ------           4.75
    1332           ------
    ----             1545
     36             2163
                   1236
                   ------
The Solidity       14.6775
```

To find the ſuperficial Content.

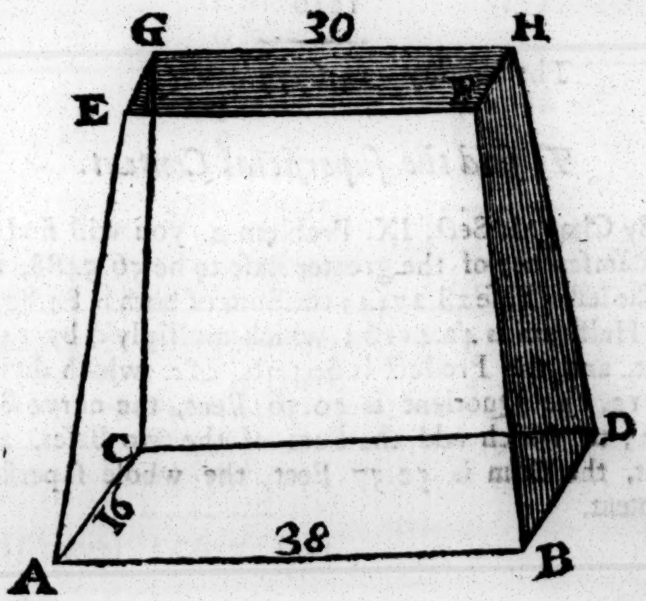
By Chap. I. Sect. IX. Problem 2. you will find the Circumference of the greater Baſe to be 56.5488, and of the leſſer Baſe 28.2744; the Sum of both is 84.8232; the Half-ſum is 42.4116; which multiply'd by 14.25 Feet, and the Product is 604.36, &c. which divided by 12, the Quotient is 50.36 Feet, the curve Surface; to which add the Sum of the two Baſes, 2.21 Feet, the Sum is 52.57 Feet, the whole ſuperficial Content.

§ IX. *To meaſure the Fruſtum of a rectangled* Pyramid, *called a* Prismoid, *whoſe Baſes are parallel one to another, but diſproportional.*

The RULE.

TO the greateſt Length add half the leſſer Length, and multiply the Sum by the Breadth of the greater Baſe, and reſerve the Product.

Then, to the leſſer Length, add half the greater Length, and multiply the Sum by the Breadth of the leſſer Baſe; and add this Product to the other Product reſerv'd, and multiply that Sum by a third Part of the Height, and the Product is the ſolid Content.



[*Example*

Example. Let ABCDEFGH be a Priſmoid given, the Length of the greater Baſe AB 38 Inches, and its Breadth AC 16 Inches; and the Length of the leſſer Baſe EF is 30 Inches, and its Breadth 12 Inches, and the Height 6 Feet; the ſolid Content is required.

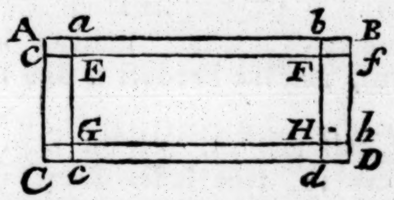
To the greater Length AB 38, add half EF the leſſer Length 15, the Sum is 53; which multiply'd by 16, the greater Breadth, and the Product is 848; which reſerve.

Again, to EF 30, add half AB 19, and the Sum is 49; which multiply by 12, (the leſſer Breadth EG) the Product is 588; to which add 848, (the reſerv'd Product) and the Sum is 1436; which multiply'd by 2, (a third Part of the Height) and the Product is 2872; divide this Product by 144, and the Quotient is 19.94 Feet, the ſolid Content.

38=AB	30=EF
15=½EF	19=½AB
—	—49
53	12=EG
16=AC	—
—	588
318	
53	
—	
848	
588	
—	
1436	
2=a third Part of the Height.	
—	
144)2872(19.94 Feet, the Content.	
—	
1432	
1360	
640	
—	
64	

To

To prove this Rule, Let us ſuppoſe the Solid cut into Pieces, ſo as to make it capable of being meaſured by the foregoing Rules, thus; let ABCD repreſent the greater Baſe, and EFGH the leſſer Baſe; and let the Solid be ſuppos'd to be cut thro' by the Lines, *ac*, *bd*, and *ef*, *gh*, from the Top to the Bottom; ſo will there be a Parallelopipedon, having its Baſes equal to the leſſer Baſe EFGH, and its Height 6 Feet, equal to the Height of the Solid: Multiply 30 (the Length of the Baſe by 12, the Breadth thereof) and the Product is 360; which multiply'd by the Height 6 Feet, and the Product is 2160. Then there are two Wedge-like Pieces, whoſe Baſes are *ab*EF, and GH*cd*; if theſe two Pieces be laid together, the thick End of one to the thin End of the other, they will compoſe a rectangled Parallelopipedon; which to meaſure, multiply the Length of the Baſe 30 by its Breadth 2, and the Product is 60; which multiply'd by 6, (the Height) the Product is 360. Then there are two other Wedge like Pieces, whoſe Baſes are *e* E *g* G, and F *f* H *h*; theſe two laid together, will compoſe a rectangled Parallelopipedon; to meaſure this, multiply the Length of the Baſe 12 by the Breadth 4, the Product is 48; which multiply'd by 6, (the Height) the Product is 288. And laſtly, there are four rectangled Pyramids, at each Corner one; which to meaſure, multiply the Length of one of the Baſes 4 by its Breadth 2, the Product is 8; which multiply'd by 2, (a third Part of the Height) the Product is 16; and that multiply'd by 4, (becauſe there are four of them) the Product is 64. Then add all theſe together, and the Sum is 2872; and divide by 144, the Quotient is 19.94 Feet, the ſame as before, which ſhews the Rule to be true.



See

See the Work.

12	30	12	4
30	2	4	2
360	60	48	8
6	6	6	2
2160	360	288	16
360			4
288			
64			64

144)2872(19.94 Feet, the whole Content.

1432
1360
640

64

To find the superficial Content.

Half the Perimeter of the greater Base is 54, and half the Perimeter of the lesser Base is 42; which added together, the Sum is 96; which multiply'd by 6, (the Height) the Product is 576: Divide this Product by 12, the Quotient is 48 Feet; to which add the Sum of the two Bases 6.72 Feet, and the Sum is 54.72 Feet, the whole superficial Content.

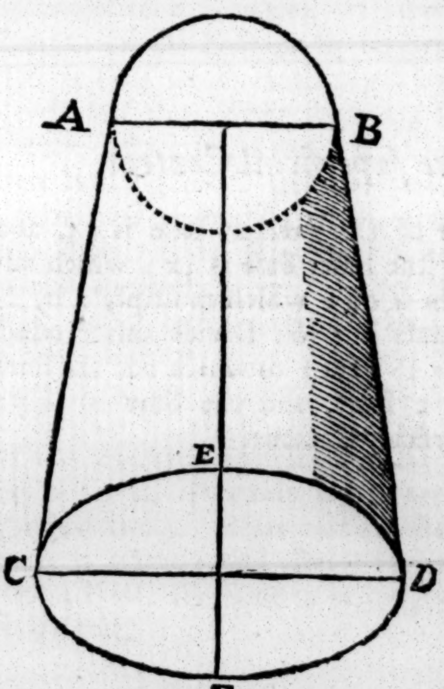
§ X.

§ X. *To meaſure a* Cylindroid; *that is, a Fruſtum of a* Cone, *having its Baſes parallel to each other, but unlike.*

The RULE.

TO the longeſt Diameter of the greater Baſe, add half the longeſt Diameter of the leſſer Baſe, and multiply the Sum by the ſhorteſt Diameter of the greater Baſe, and reſerve the Product.

Then, to the longeſt Diameter of the leſſer Baſe, add half the longeſt Diameter of the greater Baſe, and multiply the Sum by the ſhorteſt Diameter of the leſſer Baſe, and add the Product to the former reſerv'd Sum, and that Sum will be the triple Square of a mean Diameter; which multiply'd by .7854, and that Product multiply'd by a third Part of the Height, the Product is the ſolid Content.



Examp. Let ABCD be a Cylindroid, whoſe Bottom-baſe is an Oval, the tranſverſe Diameter being 44 Inches: and the conjugate Diameter 14 Inches: and the upper Baſe is a Circle, whoſe Diameter is 26 Inches, and the Height of the Fruſtum is 9 Feet; the Solidity is requir'd,

To 44 (the greater Diameter of the lower Baſe) add 13, half the

the Diameter of the greater Baſe) the Sum is 57; which multiply'd by 14, (the conjugate Diameter of the greater Baſe) the Product is 798; which reſerve. Then to 26 (the Diameter of the leſſer Baſe) add 22, (half the tranſverſe Diameter of the greater Baſe) and the Sum is 48; which multiply'd by 26, (the Diameter of the leſſer Baſe) the Product is 1248; to which add the former reſerv'd Product, the Sum is 2046; which multiply'd by .7854, the Product is 1606.9284; which multiply'd by 3, (a third Part of the Height) the Product is 4820.7852; which divided by 144, the Quotient is 33.47 Feet, the ſolid Content.

See the Work.

44=CD	26=AB
13=half AB	22=half CD
—	—
57 Sum	48 Sum.
14=EF	26=AB.
—	—
228	288
57	96
—	—
798 Product reſerv'd.	1248
	798 add.
	—
	2046
	.7854
	—
	8184
	10230
	16368
	14322
	—
	1606.9284
	3
	—

144)4820.78|52(33.47
500
687
1118
—
110

R This

This Rule being the ſame as that in the laſt Section, the Proof of that may ſerve as a ſufficient Proof of this, if what has been before written be well conſidered.

To find the ſuperficial Content.

To the Periphery of the Ellipſis 97.41, add the Periphery of the Circle 81 68, and the Sum is 179.09; the half thereof 89.545, multiply'd by 9, the Product is 805.905; which divided by 12, the Quotient is 67.16 Feet, the curve Surface: Then the Area of the Ellipſis is 3.36 Feet, and the Area of the Circle is 3.68 Feet; both which added to the curve Surface, the Sum is 74.2 Feet, the whole ſuperficial Content.

§ XI. *Of a* SPHERE *or* GLOBE.

A Sphere, or Globe, is a round ſolid Body, every Part of whoſe Surface is equally diſtant from a Point within it, called its Centre; and it may be conceiv'd to be form'd by the Revolution of a Semicircle round its Diameter. To find its Solidity, this is

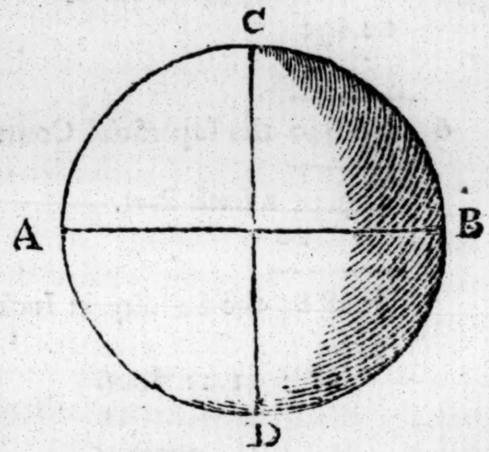
The RULE.

1. Multiply the Axis, or Diameter, into the Circumference, the Product is the ſuperficial Content; which multiply by a ſixth Part of the Axis, the Product is the Solidity.

2. Or thus: As 21 is to 11; ſo is the Cube of the Axis to the ſolid Content.

3. Or, As 1 is to .5236; ſo is the Cube of the Axis to the ſolid Content.

Example.



Example. Let ABCD be a Globe whoſe Axis is 20 Inches, then the Circumf. will be 62.832: Then, by the firſt Rule, multiply the Circumference by the Axis, and the Product will be 1256.64, which is the ſuperficial content in Inches; take a ſixth Part thereof, which is 209.44, (becauſe an exact ſixth Part of 20 cannot be taken) multiply that ſixth Part by 20 (the Axis), and the Product is 4188.8, the Solidity in Inches. Or, if you multiply the ſuperficial Content by the Axis, and take a ſixth Part of the Product, the Anſwer will be the ſame.

Or thus, by the ſecond Rule:

The Cube of the Axis is 8000, which multiply'd by 11, the Product is 88000; which divided by 21, the Quotient is 4190.47, the Solidity.

Or, by the third Rule:

If the Cube of the Axis be multiply'd by .5236, the Product is 4188.8, the Solidity, the ſame as by the firſt Way. If you divide 4188.8 by 1728, the Quotient is 2.424 Feet.

See the Work.

```
   62.832
       20
 ─────────
6)1256.640 the ſuperficial Content.
 ─────────
   209.44 a ſixth Part.
       20
 ─────────
  4188.80 the Solidity in Inches.

        21 : 11 :: 8000
                      11
                ────────
            21,88000(4190.47 the Content.
                ────────
                 40
                 190
                   100
                    160
                   ────
                     13

     1 : .5236 :: 8000
                  8000
            ──────────
1728)4188.8000(2.424 Feet, the Solidity.
     ────
      7328
       4160
         7040
         ────
          128
```

Note. If the Axis of a Globe be 1, the Solidity will be .5236; and if the Circumference be 1, the Solidity will be .016887.

By

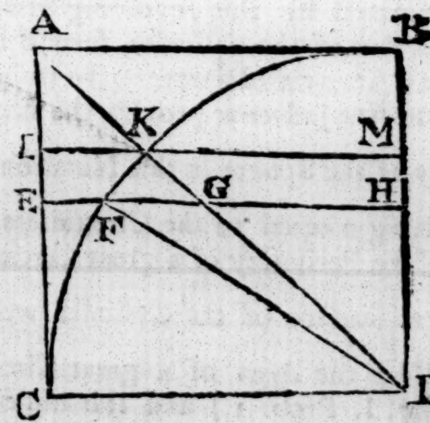
By Scale and Compaſſes.

Extend the Compaſſes from 1 to 20 (the Axis), that Extent (turn'd three times over, from .5236), will at the laſt fall upon 4188.8, the ſolid Content in Inches. Or, Extend the Compaſſes from 1728 to 8000, (the Cube of the Axis) that Extent will reach from .5236 to 2.424, the ſolid Content in Feet.

Extend the Compaſſes from 1 to 20 (the Axis), that Extent (turn'd twice over from 3.1416) will at laſt fall upon 1256.64, the ſuperficial Content in Inches: Or, Extend the Compaſſes from 144 to 400, (the Square of the Axis) that Extent will reach from 3.1416 to 8.72, the ſuperficial Content in Feet.

Demonſtration. Every Sphere is equal to a Cone, whoſe perpendicular Axis is the Radius of the Sphere, and its Baſe a Plain, equal to all the Surface of it.

For you may conceive the Sphere to conſiſt of an infinite Number of Cones, whoſe Baſes, taken all together, compoſe the Surface, and whoſe Vertexes meet all together in the Centre of the Sphere: Hence the Solidity of the Sphere will be gain'd, by multiplying its Surface by $\frac{1}{3}$ of its Radius.



Let the Square ABCD, the Quadrant CBD, and the right-angled Triangle ABD, be ſuppoſed all three to revolve round the Line BD as an Axis: Then will the Square generate a Cylinder, the Quadrant a Hemiſphere,

and the Triangle a Cone, all of the ſame Baſe and Altitude.

Then the Square of EH ($= \square$ FD) $= \square$ FH $+ \square$ DH (but DH$=$GH). And ſince Circles are as the Squares of their Diameters, (by *Eucl.* 12. 2.) the Circle made by the Revolution of EH muſt be equal to both the Circles made by the Motions of FH and GH.

If you take the Circle made by the Revolution of FH from both, there will remain the Circle made by the Motion of GH, equal to the Ring deſcrib'd by the Motion of E F. And thus it will always be, where-ever you draw the Line EH or IM, &c.

Therefore the Aggregate, or Sum, of all the Rings made by the Revolution of the EF's, muſt be equal to that of all the Circles made by the Motion of the GH's, *i. e.* the Diſh-like Solid, form'd by the revolving Rings, will be equal to the Cone, form'd by the Revolution of the GH's, which are the Elements of the Triangle ABD; that is, the Diſh-like Solid will be as the Cone, $\frac{1}{3}$ of the circumſcribing Cylinder, and conſequently the Hemiſphere muſt be $\frac{2}{3}$ of it: Wherefore the Sphere is $\frac{2}{3}$ of the circumſcribing Cylinder.

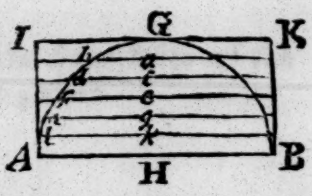
Let the Radius of the Sphere be r$=$CD, then the Diameter will be 2 r; let the Surface of the Sphere, generated by the revolving Semicircle, be called S, and that of the Cylinder, form'd by the Revolution of 2AC $=$ 2 r $=$ Diameter, be called ſ. Wherefore in what was juſt now prov'd, the Expreſſion for the Solidity of the Sphere in this Notation, will be $\frac{1\,rS}{3}$; and putting c equal to the Circumference of the Baſe, or for the Periphery of a great Circle of the Sphere, the curve Surface of the Cylinder will be 2 r c, alſo $\frac{rc}{2}$ will be the Area of a great Circle, (by Sect. IX. of Chap. I. Prob. 1.) and this multiply'd by 2r, makes r r c, which is the Solidity of the Cylinder, by Sect. V. of this Chapter. Now, ſince ſ was put equal to 2 r c $=$ the

the curve Surface of the Cylinder, $\frac{ſ\,r}{2}$ (by ſubſtituting ſ for 2 r c) will be alſo = the Solidity of the Cylinder. Now, ſince the Sphere is = $\frac{2}{3}$ of the Cylinder, $\frac{r\,S}{3} = \frac{2 \times ſ\,r}{3 \times 2}$; that is, $\frac{r\,S}{3} = \frac{2\,ſ\,r}{6} = \frac{ſ\,r}{3}$. Wherefore r S = r ſ, that is, dividing by r, S = ſ; or the Surface of the Sphere is equal to the curve Surface of the Cylinder, but the curve Surface of the Cylinder was 2 r c.

Wherefore, to find the Area of the Surface of either Sphere or Cylinder, you muſt multiply the Diameter (= 2 r) by the Circumference of a great Circle of the Sphere, or by the Periphery of the Baſe. From this Notation alſo $\frac{r\,c}{2}$, the Area of a great Circle of the Sphere is plainly $\frac{1}{4}$ of 2 r c, the Surface of the Sphere; that is, the Surface of the Sphere is Quadruple of the Area of the greateſt Circle of it.

Wherefore, to 2 r c, the convex Surface of the Cylinder, add r c, equal to the Area of both its Baſes, you will have 3 r c; which ſhews you, that the Surface of the Cylinder (including its Caſes) is to the Surface of the Sphere as 3 to 2; or that the Sphere is $\frac{2}{3}$ of the circumſcribing Cylinder, in Area as well as Solidity.

Or you may prove the Sphere to be $\frac{2}{3}$ of the Cylinder of the ſame Baſe and Altitude, by Lemma VI. aforegoing, thus:



Let A G B repreſent the Hemiſphere, and A I K B half the Cylinder; then, if the Semidiameter G H be divided into ſix equal Parts, and Lines drawn parallel to A B, the Diameter, the Squares of the Semichords, ab, cd, ef, &c. will be

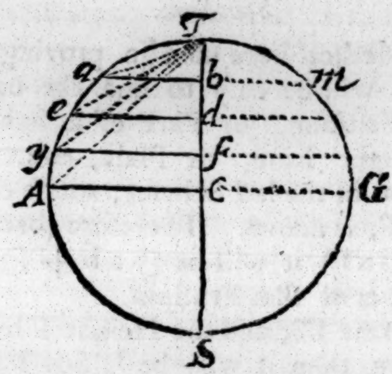
be a Series of Numbers, whose greatest Term AH is a square Number, the other differing by odd Numbers; that is, AH is 36, k l 35, g h 32, e f 27, c d 20, a b 11: But an infinite Series of such Numbers are in Proportion to an infinite Number of Terms equal to the greatest, as 2 to 3. And because the Hemisphere is compos'd of an infinite Number of Circles, whose Diameters are the Chords of the Semicircle; and the Half-cylinder is compos'd of an infinite Number of Circles, whose Diameters are all equal to the Diameter of the Semicircles A B; therefore the Hemisphere is in Proportion to the Half-cylinder, as 2 to 3; and consequently, the whole Sphere bears the same Proportion to the whole Cylinder.

That the Superficies of every Sphere (or Globe) is equal to four times the Area of its greatest Circle, is thus proved:

The Solidity of the Sphere is constituted of an infinite Number of parallel Circles (as is aforesaid); consequently, the Superficies of the Sphere will be compos'd of the Peripheries of those Circles which constitute its Solidity.

Note, In the following Demonstrations, ⊙ signifies any Circle in general; and if any two Letters be joined to it, thus, ⊙ A B, &c. then it denotes the Area of such a Circle as those two Letters represent the Radius of.

Let



Let $D = TS$, the Axis of any Sphere; then, according to the Property of a Circle, it

will be	1	$\overline{D-Tb} \times Tb = \square ab$;
that is,	2	$D \times Tb - \square Tb = \square ab$;
therefore	3	$D \times Tb = \square aT$.
For	4	$\square ab + \square Tb = \square aT$ (*Eucl.* 1. 47.)
and	5	$D \times Td = \square eT$.
	6	$D \times Tf = \square yT$.

Hence it is evident, that the Series $\square aT$, $\square eT$, $\square yT$, *&c.* are in the ſame Ratio with Tb, Td, Tf, *&c. viz.* in arithmetical Progreſſion: Whence it follows, that the $\odot$ *a*T = to the Sum of all the Circle's Peripheries, between T and b.

And $\odot$ eT = the Sum of all the Circle's Peripheries between T and d, *&c.*

Conſequently, that the $\odot$ AT = the Sum of all the Circle's Peripheries, included between T and C; that is, $\odot$ AT = the Superficies of the Hemiſphere.

And becauſe $\square AC + \square TC = \square AT$, and $\square AC$ is equal to $\square TC$; therefore $\odot AT = 2 \odot AC$, is the Superficies of the Hemiſphere.

Conſequently, $4 \odot AC$ will be the Superficies of the whole Sphere. Which was to be proved.

Scholium

Scholium.

From the Method here uſed in proving the whole Superficies, it will be eaſy to find the curve Superficies of any Fruſtum, or Part of a Sphere, that is cut off by a right Line, or Plain, *viz.* ſuch as the Fruſtum a T m in the laſt Scheme, whoſe curve Superficies is $\ominus$ a T, as above. Therefore (becauſe $\square$ a b $+ \square$ T b $= \square$ aT) it will be $\bigcirc$ a b $+ \odot$ T b $=$ the curve Superficies of that Fruſtum.

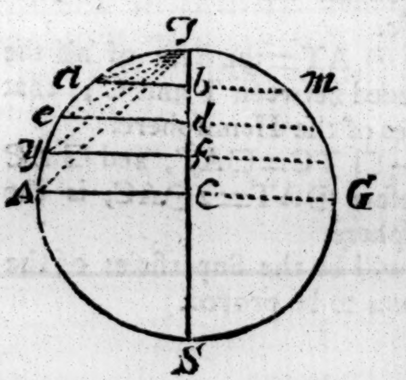
But if the Axis TS, and the Height T b of the Fruſtum, are given, then it will be T S $\times$ T b $= \square$ a T, as in the third Step above, which gives the Proportion or Theorem following, *viz.*

As the Axis of the Sphere is to the whole Superficies of the Sphere; ſo is the Height of any Fruſtum to its curve Superficies.

To which if there be added the Area of the Fruſtum's Baſe, the Sum will be the whole Superficies of the Fruſtum.

That the Solidity of every Sphere is Two-thirds of its circumſcribing Cylinder, *may be thus proved.*

According to the Work above, it appears, that $\odot$ a b, $\odot$ e d, $\odot$ y f, &c. do conſtitute the Solidity of the Sphere; and that $\square$ a T, $\square$ e T, $\square$ y T, &c are a Series of Terms in Arithmetical Progreſſion, $\square$ A T being the greateſt Term, and T C the Number of Terms; therefore $\ominus$ AT $\times \frac{1}{2}$ TC $=$ the Sum of all the Series, by Lemma 2.



And becauſe $\square$ aT $- \square$ Tb $= \square$ a b. $\square$ eT $- \square$ Td $= \square$ e d. $\square$ y T $- \square$ T f $= \square$ y f. $\square$ AT $- \square$ TC $= \square$ A C, &c. wherein

wherein □ T b, □ T d, □ T f, &c. are a Series of Squares, whose Roots T b, T d, T f, are in arithmetical Progression; □ TC being the greatest Term, and TC the Number of Terms; therefore ⊙ T C × $\frac{1}{3}$ TC = the Sum of all the Series, by Lemma 3.

Consequently, ⊙ AT × $\frac{1}{2}$ TC — ⊙ TC × $\frac{1}{3}$ TC = the Sum of all the Series ⊙ ab, ⊙ ed, ⊙ y f, &c. which constitute the Solidity of the Half-sphere ATG. Put D = 2 TC the Axis of the Sphere; then $\frac{1}{3}$ D = $\frac{1}{2}$ TC, and $\frac{1}{6}$ P = $\frac{1}{3}$ TC. And because □ AT = 2 □ TC, therefore ⊙ AT = 2 ⊙ TC = 1.5708 DD; and 1.5708 DD × $\frac{1}{4}$ D = 0.3927 DDD.

Again, ⊙ TC × $\frac{1}{3}$ TC = 0.7854 DD × $\frac{1}{6}$ D = .1309 DDD, then 0.3927 DDD — 0.1309 DDD = 0.2618 DDD, the Solidity of the Half-sphere.

Consequently, 0.2618 DDD × 2 = .5236 DDD, will be the solid Content of the whole Sphere, which is equal to $\frac{2}{3}$ of the Cylinder; the Diameter of whose Base, and its Height, is = D.

For 0.7854 DDD = the Solidity of the Cylinder by Sect. V. But $\frac{2}{3}$ of 0.7854 DDD = 0.5236 DDD, as before.

Scholium.

From this Demonstration it will be easy to deduce or raise Theorems for finding the solid Content of any Frustum of a Sphere, as T m in the last Figure.

For we there suppose the Frustum a T m to be constituted of an infinite Series of Circles, which have the same Ratio with all those Circles that constitute the Half-sphere.

Therefore it follows, that ⊙ a T × $\frac{1}{2}$ Tb : — ⊙ bT × $\frac{1}{3}$ T b will be the Sum of all the Circles intercepted between T and b; consequently it will be the Solidity of that Frustum.

And because □ a b + □ Tb = aT; therefore ⊙ ab + ⊙ T b × $\frac{1}{2}$ Tb : — ⊙ Tb × $\frac{1}{3}$ T b = the Solidity. Let c = a b half the Diameter of the Frustum's Base h = Tb its Height; and S = the Solidity of the Frustum. Then ⊙ ab = 3.1416cc, and ⊙ Tb = 3.1416hh; consequently,

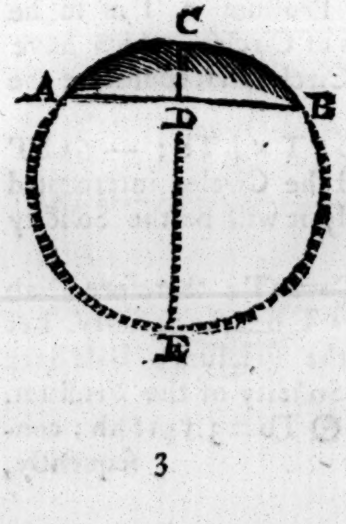
ſequently, $\frac{3.1416cch + 3.1416hhh}{2} - \frac{3.1416hhh}{3} = S$.

Which being reduced, will become $3\,c\,c\,h + h\,h\,h \times 0.5236 = S$; which is one Theorem for finding the Solidity of the Fruſtum, and may be expreſſed in Words; thus:

If to three times the Square of the Semidiameter of the Fruſtum's Baſe, you add the Square of the Height of the Fruſtum, and multiply the Sum by the Height of the Fruſtum, and that Product multiply'd by .5236, the Product will be the ſolid Content.

But if the Axis of the Sphere, and the Height of the Fruſtum, be given; then put $D =$ the Axis, $h =$ the Height of the Fruſtum, and c as before; it will be $D - h \times h = c\,c$, *viz.* $D\,h - h\,h = c\,c$. Then will $3\,D\,h\,h - 2\,h\,h\,h = 3\,c\,c\,h + hhh$; conſequently, $3\,Dhh - 2hhh \times 0.5236 = S$, the Fruſtum's Solidity. Which is another Theorem for finding the Solidity of the Fruſtum, and may be expreſſed in Words thus:

From three times the Axis ſubtract twice the Height of the Fruſtum, and multiply the Remainder by the Square of the Height, and that Product multiply by .5236, this laſt Product will be the Solidity of the Fruſtum.



Example. Let ABCD be the Fruſtum of a Sphere; ſuppoſe AB (the Diameter of the Fruſtum's Baſe) be 16 Inches, and CD (the Height) 4 Inches; the Solidity is required.

By the first Rule.

8
8
—
64 Square of the Semidiameter AD.
3
—
192
16 add the Square of CD.
—
208
4 multiply by CD.
—
832

.5236
832
10472
15708
41888
435.6352

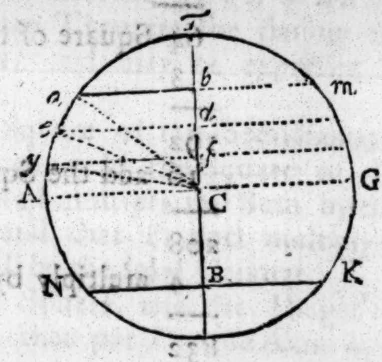
By the second Rule, thus:

First, by the Rule in Page 113, you will find the Axis of the whole Globe to be 20 Inches.

	20 Axis	.5236	
	3	832	
From	60	10472	
Subtr.	8 twice CD.	15708	
		41888	
Rem.	52		the solid Content,
Mult.	16 Sq. of CD.	435.6352	the same as before.
	312		
	52		
Prod.	832		

And if it be requir'd to find the middle Part, amNK, uſually call'd the middle Zone of a Sphere,

Then, becauſe it is ſuppos'd that a m = NK, or (which is all one) that b C = C B; therefore it is plain, that if twice the Segment, aTm, be taken from the whole Sphere, there will remain the middle Zone amNK.



But becauſe that Work is a little troubleſome, I will here ſhew how to raiſe a Theorem for the doing it,

Firſt, becauſe AC = yC = eC = aC = TC, therefore it will be □ A C — □ C f = y f, □ A C — □ C d = □ ed, □ A C — □ C b = □ a b, &c.

Here, becauſe □ A C, □ A C, □ A C, &c. are a Series of Equals, and C b the Number of all the Terms; therefore □ A C × C b = the Sum of all that Series, *per* Lemma I.

And □ C f, □ C d, □ C b, &c. being a Series of Squares, whoſe Roots are in arithmetical Progreſſion, beginning at the Centre, C, *viz.* o, C f, C d, C b, &c. wherein the greateſt Term is □ Cb, and the Number of Terms is Cb; therefore □ C b × $\frac{1}{3}$ C b = the Sum of all the Series, *per* Lemma III.

Conſequently, the ⊙ AC × C b :— ⊙ C b × $\frac{1}{3}$ C b = the Sum of all the Series ⊙ y f, ⊙ e d, ⊙ a b, &c. which do chnſtitute the Solidity of the half Zone a m A G.

And

And becauſe $\square AC - \square Cb = \square ab$, therefore $\odot AC - \odot ab = \odot Cb$. Conſequently $\dfrac{\odot AC \times Cb : - \odot AC + \odot ab : \times Cb}{3} = 2 \odot AC + \odot ab : \times \frac{1}{3} Cb$ will be Solidity of the half Zone.

Put $D = AG = 2AC$, $x = am$, and $H = bB = 2Cb$. Then $\odot AC = .7854\, DD$, $\odot ab = .7854\, xx$. And if we turn the common Factor .7854 into a Diviſor 1.27323, and then take the Triple of that Diviſor, *viz.* 3.8197, the Reſult of the precedent Work will produce this following Theorem.

$$\text{Theo.} \left\{ \frac{2DD + xx}{3.8197} : \times H = \right\} \text{the middle Zone } amNK.$$

Which in Words is thus: To twice the Square of the Axis AG, and the Square of the Diameter of the Fruſtum's Baſe (am), and divide the Sum by 3.8197, then multiply the Quotient by the Height or Thickneſs of the middle Zone, and the Product will be the Solidity of the middle Zone required.

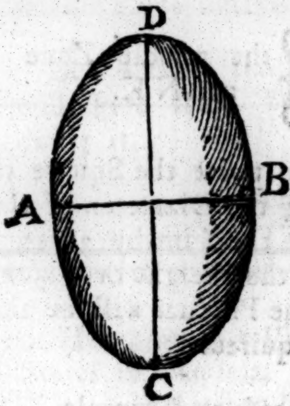
This is ſo plain and eaſy, it needs no Example.

§ XII. *Of a* SPHEROID.

A Spheroid is a Solid reſembling an Egg. To find the ſolid Content thereof, this is

The RULE.

Multiply the Square of the Diameter ef the greateſt Circle by the Length, and that Product multiply again by .5236; this laſt Product will be the Solidity of the Spheroid.

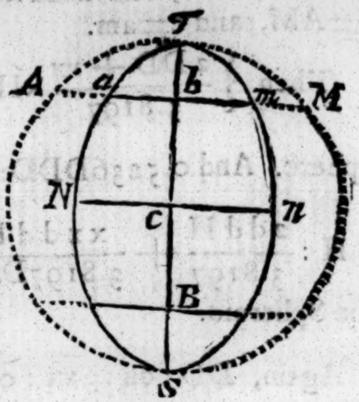


Let AB, the Diameter of the greateſt Circle, be 33 Inches, and CD (the Length) 55 Inches; the Solidity is requir'd.

33	59895
33	.5236
99	359370
99	179685
1089	119790
55	299475
5445	31361.0220 the Solidity.
5445	
59895	

Demon-

Demonftration. Every Spheroid is equal to $\frac{2}{3}$ of a Cylinder, whofe Bafe is equal to the greateft Circle of the Spheroid, and its Height equal to the Length of the Spheroid.



Suppofe the Figure NTnSN in the annex'd Scheme, to reprefent a Spheroid, form'd by the Rotation of the Semi-Ellipfis TNS, about its tranfverfe Axis TS.

Let D = T S, the Length of the Spheroid, and the Axis of its circumfcribing Sphere; and d = N n, the Diameter of the greateft Circle of the Spheroid.

Then becaufe, □T C : □NC : : □Ab : □ a b, by *Sect.* XV. *Step.* 3. *Page* 125.

Therefore it will be, D D : d d : : □ A b : □ a b.

But the Sum of an infinite Series of fuch Circles as ⊙ A b (whofe Diameters are Chords) do conftitute the Solidity of the Sphere. (By *Sect.* XI.)

And the Sum of an infinite Series of fuch Circles as ⊙ a b (*viz.* whofe Diameters are Ordinates of the Ellipfis) do conftitute the Solidity of the Spheroid.

Therefore, DD : dd : : 0.5236 DDD : 0.5236 Ddd = the Solidity of the Spheroid. (*Eucl.* 5. 12.)

But 0.5236 Ddd = $\frac{2}{3}$ of the Cylinder, whofe Diameter is = d, and Height = D. (By *Sect.* V.)

Now, from this Proportion between the Sphere and its infcrib'd Spheroid, it will be very eafy to deduce Theorems for finding the folid Content, either of the Fruftum or middle Zone of any Spheroid; having the fame Height with that of the Sphere; for,

As the Solidity of the whole Sphere is to the Solidity of the whole Spheroid; fo is any Part of the Sphere to the like Part of the Spheroid.

As for Inſtance: Suppoſe it was requir'd to find the middle Zone of any Spheroid.

Let D=TS, and d=Nn, as above; and H=bB, x=AM, and c=am.

Then $\left\{\frac{2DD+xx}{3.8197}\right. \times H =$ the middle Zone of the Sphere. And 0.5236DDD: 0.5236ddD :: $\frac{2DD+xx}{3.8197} \times H : \frac{2ddH}{3.8197} + \frac{xxddH}{3.8197DD} =$ the middle Zone of the Spheroid.

Again, DD : dd :: xx : cc. Therefore $\frac{xxdd}{DD} = cc$.

Conſequently, $\frac{xxdd}{DD} \times \frac{H}{3.8197} = \frac{cc}{3.8197} \times H$. Which being taken inſtead of $\frac{xxddH}{3.8197DD}$, there will ariſe this following Theorem $\left\{\frac{2dd+cc}{3.8197}\right. : \times H =$ the middle Zone of the Spheroid.

Note, That 3.8197 = 1.2732 × 3. See *Page* 102.

§ XIII. *Of a Parabolic* CONOID.

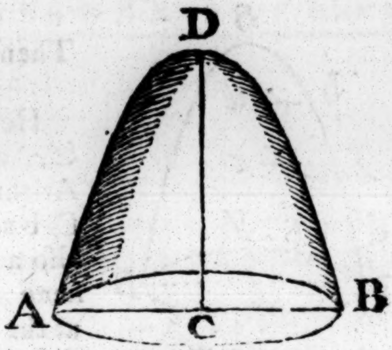
A Parabolic Conoid is ſomething like an half Spheroid, having its Sides ſomewhat ſtraighter. It is generated by ſuppoſing a Semi-parabola turn'd about its Axis. To find the ſolid Content thereof, this is

The RULE.

Multiply the Square of the Diameter of its Baſe by .7854, and multiply that Product by half the Height, that laſt Product ſhall be the ſolid Content.

Let

Let ABCD be a Parabolic Conoid, the Diameter of whoſe Baſe is 36 Inches, and its Height CD 33 Inches; the Solidity is requir'd.

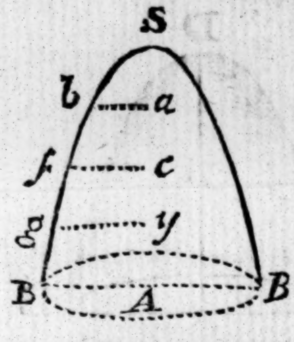


36	.7854	1017.8784
36	1296	33
216	47124	30536352
108	70686	30536352
	15708	
1296	7854	2)33589.9872
	1017.8784	16794.9936

1728)16794.99|36(9.71 Feet the Content.
15552
12429
12096
3339
1728
1611

Demonſtration. The Parabolic Conoid is conſtituted of an infinite Number of Circles, whoſe Diameters are the Ordinates of the Parabola. Now according to the Property of every Parabola, it will be, $SA : AB :: AB : \frac{\square AB}{SA} = L.$ the *Latus Rectum.*

Then



Then $\begin{cases} Sa \times L = \square\, ba, \\ Se \times L = \square\, fe, \\ Sy \times L = \square\, gy, \text{ \&c.} \end{cases}$

Here $Sa \times L$, $Se \times L$, $Sy \times L$, &c. are a Series of Terms in Arithmet. Progreſſ. Therefore $\square\, ba$, $\square\, fe$, $\square\, gy$, &c. are alſo a Series of Terms in the ſame Progreſſion, beginning at the Point S, wherein $\square\, AB$ is the greateſt Term, and S A the Number of all the Terms. Therefore $\square\, AB \times \frac{1}{2} SA =$ the Sum of all the Series. (By Lemma 2.)

Conſequently, $\odot\, AB \times \frac{1}{2} SA =$ the Sum of all the Series of $\odot\, ba$, $\odot\, fe$, $\odot\, gy$, &c. which do conſtitute the Solidity of the Conoid.

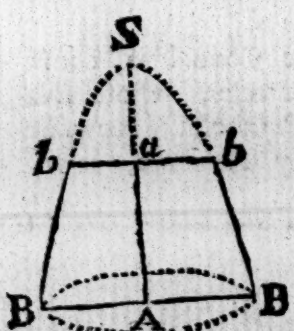
Put $D = 2\,AB$, and $H = SA$.

Then $.7854\, DD \times \frac{1}{2} H = .3927\, DDH$ will be the ſolid Content of the Conoid; which is juſt half the Cylinder, whoſe Baſe is $= D$, and Height $= H$.

This being rightly underſtood, it will be eaſy to raiſe a Theorem for finding the lower Fruſtum of any Parabolic Conoid.

For, ſuppoſing $h = aA$, the Height of the Fruſtum, and $p = Sa$, the Height of the Part bSb cut off; and $h + p = SA$, the Height of the whole Conoid.

Conſequently, $\dfrac{\odot\, AB \times h + \odot\, AB \times p}{2} =$ the Solidity of the whole Conoid.



And $\dfrac{\odot\, ba \times p}{2} =$ the Solidity of the Part cut off.

Therefore

Therefore	1	$\frac{\ominus AB\times h+\odot AB\times p-\bigcirc ba\times p}{2}$ The Solidity of the Frustum.
But	2	$h+p:\square AB::p:\square ba$
Consequ.	3	$h+p:\bigcirc AB::p:\odot ba$
$3 \because$	4	$\odot AB\times p:=\ominus ba\times ph+\bigcirc ba\times p$
$4-\bigcirc ba\times p$	5	$\odot AB\times p:=\odot ba\times h+\bigcirc ba\times h$
1×2	6	$\bigcirc AB\times h:+\bigcirc AB\times p:-\bigcirc ba\times p=2F$.
$6-5$	7	$\bigcirc AB\times h=2F:-\odot ba\times h$.
$7+\odot ba\times h$	8	$\odot AB\times h:+\odot ba\times h=2F$.
$8\div 2$	9	$\frac{\bigcirc AB+\bigcirc ba}{2}:\times h=F$, the Frustum's Solidity.

Let D = 2 A B, as before, and d = 2 b a, the Diameter of the Part cut off; then we shall have this following Theorem.

$0.3927\ DD+0.3927\ dd:\times h=$ the Solidity of the Frustum requir'd: Which in Words is thus:

Multiply the Sum of the Squares of the greater and lesser Diameters by .3927, and that Product by the Height of the Frustum, the last Product shall be the solid Content.

§ XIV. *Of a Parabolic* SPINDLE.

IF an acute Parabola be suppos'd to be mov'd about its greatest Ordinate, it will form a Solid, call'd a Parabolic Spindle. To find the solid Content, this is

The

The RULE.

Multiply the Square of the Diameter of its greatest Circle by .41888, (being $\frac{8}{15}$ of .7854) and that Product by its Length; that last Product is the solid Content.

Let ABCD be a Parabolic Spindle, whose greatest Diameter CD is 36 Inches, and its Length AB 99 Inches; the Solidity is requir'd.

```
  36=CD        .41888
  36             1296
 ----          -------
  216          251328
 108          376992
 ----          83776
 1296 Square  41888
              -------
              542.86840
                     99
              ---------
             488581632
            488581632
           ----------
 1728)53743.97952(31.10184
        . . . . . .
       ---
       1903
        1759
          3179
          14515
           6912
          -----
          . . . .
```

The solid Content is 31.10184 Feet.

Demonſtration. A Parabolic Spindle is conſtituted of an infinite Series of Circles, whoſe Diameters are all parallel to the Axis of the Parabola, as $\odot$ m a, $\odot$ n e, $\odot$ p y, *&c.*

Let us ſuppoſe the Line S d parallel to AB, *&c.* Then it hath already been prov'd, that the Lines f m, g n, h p, *&c.* are a Series of Squares, whoſe Roots are in arithmetical Progreſſion, conſequently their Squares, *viz.* $\square$ f m, $\square$ g n, $\square$ h p, *&c.* will be a Series of Biquadrats, whoſe Roots will be arithmetical Progreſſion: Which being premis'd, we may proceed thus:

First { 1 | S A − f m = m a.
2 | S A − g n = n e.
3 | S A − h p = p y.

1 ☉ 2 | 4 | $\square$ S A − 2 S A × f m + $\square$ f m = $\square$ m a.
2 ☉ 2 | 5 | $\square$ S A − 2 S A × g n + $\square$ g n = $\square$ n e.
3 ☉ 2 | 6 | $\square$ S A − 2 S A × h p + $\square$ h p = $\square$ p y, *&c.*

1. In theſe Equations, the $\square$ S A, $\square$ S A, $\square$ S A, being a Series of Equals, and A B the Number of all the Terms; therefore it will be $\square$ S A × A B = the Sum of all the Series, by Lemma I.

2. Becauſe f m, g n, h p, *&c.* are a Series of Squares, wherein S A is the greateſt Term, and A B the Number of all the Terms;

Therefore $\frac{2\,SA \times SA \times AB}{3} = \frac{2\,\square SA \times AB}{3}$ will be the Sum of all the Series, by Lemma III.

3. And the $\square$ f m, $\square$ g n, $\square$ h p, &c. will be a Series of Terms in the Ratio of Biquadrats as above, $\square$SA being the greateſt Term, and AB the Number of all the Terms. Therefore it will be $\frac{\square SA \times AB}{5}$ = the Sum of all the Series, by Lemma V.

Whence it follows, that $\square SA \times AB - \frac{2 \square SA \times AB}{3} + \frac{\square SA \times AB}{5}$ = the Sum of all the Series of $\square$ ma, $\square$ n e, $\square$ p y, &c.

That is, $\frac{8 \square SA \times AB}{15}$ = the Sum of all the Series, $\square$ m a, $\square$ n e, $\square$ p y, &c. Conſequenty, $\frac{8 \bigcirc SA \times AB}{15}$ = the Sum of all the Series of Circles, $\odot$ m a, $\bigcirc$ n e, $\bigcirc$ p y, &c. which conſtitute the Solidity of half the Spindle, *viz.* of S A B.

Therefore putting D = 2 S A, and H = 2 A B, it will be 0.41888 D D H = the Solidity of the whole Parabolic Spindle b S B, being $\frac{8}{15}$ of 0.7854 D D H, the Solidity of its circumſcribing Cylinder.

From hence we may alſo raiſe a Theorem for finding the Fruſtum, S A p y, of the laſt Figure.

For $\odot$ S A being the greateſt Term, $\odot$ p y the leaſt Term, and A y the Number of all the Terms or Circles included between A and y :

Therefore

Therefore | 1 | $\square SA - \frac{2\,SA \times hp}{3} + \frac{\square hp}{5} : \times Ay = z$ the Sum of all the Series, $\square$ S A, $\square$ ma, $\square$ en, $\square$ py,

$1 \times \overline{3}$ | 2 | $3\,\square SA - 2SA \times hp + \frac{3\square hp}{5} : \times Ay = 3\overline{z}$

$z - Ay$ | 3 | $3\,\square SA - 2\,SA \times hp + \frac{3\square hp}{5} = \frac{3\,z}{Ay}$

But | 4 | $\square SA - 2SA \times hp = \square py - \square hp$, *per St.* 6.

$3 - 4$ | 5 | $2\,\square SA + \frac{\square 3hp}{5} = \frac{3z}{Ay} - \square py + \square hp.$

$\overline{5} + $ &c. | 6 | $2\square SA + \square py - \frac{2}{5}\square hp = \frac{3z}{Ay}$

Conſeq. | 7 | $2 \odot SA + \odot py - \frac{2}{5} \odot hp : \times \frac{1}{3}\,Ay = z$ the Sum of all the Series of $\odot$ SA, $\odot$ ma, $\odot$ne, $\odot$py, which do conſtitute the Solidity of the Fruſtum SApy. Therefore, putting $D = 2\,SA$, as before, $C = 2\,py$, $x = 2hp$, and $H = Ay$; it will be $1.5708\ DD + .7854\ CC - .31416\ xx : \times \frac{1}{3}\ H =$ the Fruſtum SApy. And if we make $L = 2H$, then $1.5708\ DD + .7854CC - .31416\ xx : \times \frac{1}{6}\ L =$ the Double of that Fruſtum, being the middle Zone. Which in Words is thus:

Multiply the Square of the greateſt Diameter by 1.5708, and multiply the Square of the leſſer Diameter by .7854, and multiply the Square of the Difference of the Diameters by .31416; from the Sum of the two former Products ſubtract the latter Product, and multiply the Remainder by one third Part of the Length, and that Product will be the Solidity of the middle Zone required.

CHAP. III.

Of the Meaſuring the Works of the ſeveral Artificers relating to Building; and what Methods and Cuſtoms are obſerved therein.

§ I. *Of* CARPENTERS *Work.*

THE Carpenters Work which are meaſurable, are Flooring, Partitioning, and Roofing; all which are meaſured by the Square of 10 Feet long, and 10 Feet broad; ſo that one Square contains 100 ſquare Feet.

1. *Of Flooring.*

If a Floor be 57 Feet 3 Inches long, and 28 Feet 6 Inches broad; How many Squares of Flooring are there in that Room?

Multiply 57 Feet 3 Inches by 28 Feet 6 Inches, and the Product is 1631 Feet, *&c.* which divide by 100 (this is done by cutting from the Product two Figures towards the Right-hand with a Daſh of the Pen)

Pen); and the remaining Figures are the Quotient, and the Figures cut off are Feet: Thus, 1631 divided by 100, by cutting off 31 from the Right-hand thereof, the Quotient is 16 Squares, and the 31 cut off is 31 Feet.

See the Work, both by Decimals, and alſo by Feet and Inches.

57.25	F.	I.	
28.5	57	3	
28625	28	6	
45800	456		
11450	114		
16|31.625	28	7	6
	7	0	0
	16|31	7	6

Facit 16 Squares and 31 Feet.

Note, That .5 is the Decimal for half of any thing, .25 is the Decimal for a Quarter, and .125 is the Decimal for half a Quarter; ſo in the laſt Example, .25 is the Decimal of 3 Inches, becauſe 3 Inches is a Quarter of a Foot; and .5 is the Decimal of 6 Inches, becauſe 6 Inches is half a Foot.

Example 2. Let a Floor be 53 Feet 6 Inches long, and 47 Feet 9 Inches broad, How many Squares are contain'd in that Floor?

47.75	F.	I.	
53.5	53	6	
23875	47	9	
14325	371		
23875	212		
25|54.625	26	9	
	13	4	6
	23	6	
	25|54	7	6

Facit 25 Squares and 54 Feet.

T 2 By

By Scale and Compaſſes.

In the firſt Example, extend the Compaſſes from 1 to 28.5, that Extent will reach from 57.25 to 16 Squares and near a third Part.

In the ſecond Example, extend the Compaſſes from 1 to 47.5, that Extent will reach from 53.5 to 25 Squares and above an Half.

2. *Of Partitioning.*

Example 1. If a Partition between Rooms be in Length 82 Feet 6 Inches, and in Height 12 Feet 3 Inches; How many Squares are contained therein?

The Length and Breadth being multiply'd together, the Product is 1010.625; which divide by 100, (as before is ſhew'd) and the Anſwer is 10 Squares 10 Feet; the Inches or Parts, in theſe Caſes, are of no Value.

<table>
<tr><td>12.25</td><td>F.</td><td>I.</td><td></td></tr>
<tr><td>82.5</td><td>82</td><td>6</td><td></td></tr>
<tr><td></td><td>12</td><td>3</td><td></td></tr>
<tr><td>6125</td><td></td><td></td><td></td></tr>
<tr><td>2450</td><td>990</td><td>0</td><td></td></tr>
<tr><td>9800</td><td>20</td><td>7</td><td>6</td></tr>
<tr><td>10|10.625</td><td>10|10</td><td>7</td><td>6</td></tr>
</table>

Facit 10 Squares 10 Feet.

Example 2. If a Partition between Rooms be in Length 91 Feet 9 Inches, and in Breadth 11 Feet 3 Inches; How many Squares are contain'd therein?

The Length and Breadth being multiply'd together, the Product is 1032 Feet; which divided by 100, the Anſwer will be 10 Squares and 32 Feet.

91.75

91.75		F,	I.	
11.25		91	9	
		11	3	
45875				
18350		1009	3	
9175		22	11	3
9175				
10,32.1875		10|32	2	3

3. *Of Roofing.*

It is a Rule amongst Workmen, that the Flat of any House, and half the Flat thereof, taken within the Walls, is equal to the Measure of the Roof of the same House; but this is when the Roof is true pitched: For if the Roof be more flat or steep than the true Pitch, it will measure to more or less accordingly.

Example 1. If a House within the Walls be 44 Feet 6 Inches long, and 18 Feet 3 Inches broad; How many Squares of Roofing will cover that House?

Multiply the Length and Breadth together, and the Product is 812 Feet, the Flat; the half thereof is 406 Feet; which added to the Flat, the Sum is 1218 Feet; which divided by 100, the Answer is 12 Squares aud 18 Feet.

	18.25
	44.5
	9125
	7300
	7300
Flat	812.125
Half	406
	12.18

	F.	I.	
	44	6	
	18	3	
	352		
	44		
	11	1	6
	9	0	0
The Flat	812	1	6
The Half	406		
Sum	12|18		

Facit 12 Squares 18 Feet.

By Scale and Compaſſes.

In the firſt Example of Partitioning, extend the Compaſſes from 1 to 12.25, that Extent will reach from 82.5 to 10 Squares and one Tenth.

In the ſecond Example, extend the Compaſſes from 1 to 11.25, that Extent will reach from 91.75 to 10 Squares, and a little leſs than a third Part.

In the Example of Roofing, extend the Compaſſes from 1 to 18.25, that Extent will reach from 44.5 to 812 the Flat; to which add the Half thereof, and the Sum is 12.18, which is 12 Squares 18 Feet, as above.

There are other Works about a Building, done by the Carpenter, that are meaſured by the Foot, running Meaſure, that is, by the Number of Feet in Length only; as Cornices, Doors and Caſes, Window-frames, Guttering, Lintels, Sommers, Skirt-boards, &c.

Note 1. In the meaſuring of Flooring, after you have meaſured the whole Floor, you muſt deduct out of it the Well-holes for the Stairs and Chimneys; and in Partitioning, for the Doors, Windows, &c. except (by Agreement) they are to be included.

Note 2. In meaſuring of Roofing, ſeldom any Reductions are made for the Holes for the Chimney-ſhafts, the Vacancies for Lutheren-lights and Sky-lights; for they are more Trouble to the Workman, than the Stuff which would cover them is worth.

§ II. *Of* BRICKLAYERS *Work*.

THE principal is Tiling, Walling, and Chimney-work.

1. *Of Tiling*.

Tiling is meaſured by the Square of 100 Feet, as Flooring, Partitioning and Roofing were in the Carpenters Work; ſo that between the Roofing and Tileing, the Difference will not be much; yet the Tiling will be the moſt; for the Bricklayers ſometimes will require to have double Meaſure for Hips and Vallies. When Gutters are allowed double Meaſure, the way is to meaſure the Length along the Ridge-tile, and by that means the Meaſure of the Gutters becomes double; it is uſual alſo to allow double Meaſure at the Eaves, ſo much as the Projector is over the Plate, which is commonly about 18 or 20 Inches.

Example 1. There is a Roof covered with Tiles, whoſe Depth on both Sides (with the uſual Allowance at the Eaves) is 37 Feet 3 Inches, and the Length 45 Feet; I demand how many Squares of Tiling are contained therein?

F.	I.		37.25
37	3		45
45	0		18625
185			14900
148			16,762.5
11	3		
16|76	3		

Anfwer, 16 Squares 76 Feet.

Example 2. There is a Roof covered with Tiles, whofe Depth on both Sides (with the Allowance at the Eaves) is 35 Feet 9 inches, and the Length 43 Feet 6 Inches; I demand how many Squares of Tileing are in the Roof?

F.	I.			35.75
43	6			43.5
35	9			17875
215				10725
129				14300
21	9			15|55.125
30	10	6		
17	6			
15|55	1	6		

Here the Length aud Depth being multiply'd together, the Product is 1555 Feet; which divided by 100, (as before is taught) the Anfwer is 15 Squares and 55 Feet.

By Scale and Compaffes.

In the firft Example, extend the Compaffes from 1 to 37.25, that Extent will reach from 45 to 16 Squares and a little above three Quarters of a Square.

In

In the ſecond Example, extend the Compaſſes from 1 to 35.75, that Extent will reach from 43.5 to 15 Squares and 55 Feet, that is, a little above a Half-ſquare.

2. *Of Walling.*

Bricklayers commonly meaſure their Work by the Rod ſquare of 16 Feet and a half; ſo that one Rod in Length and 1 in Breadth, contain 272.25 ſquare Feet; for 16.5, multiply'd in itſelf, produces 272.25 ſquare Feet. But in ſome Places the Cuſtom is to allow 18 Feet to the Rod; that is, 324 ſquare Feet. And in ſome Places the uſual Way is, to meaſure by the Rod of 21 Feet long and 3 Feet high, that is, 63 ſquare Feet; and here they never regard the Thickneſs of the Wall, but the uſual Way is to moderate the Price according to the Thickneſs.

When you meaſure a Piece of Brick-work, the firſt thing is to inquire by which of thoſe Ways it muſt be meaſured; then, having multiply'd the Length and Breadth in Feet together, divide the Product by the proper Diviſor, either for Rods or Roods, and the Quotient is ſquare Rods, or ſquare Roods. accordingly.

But cemmonly Brick-walls, that are meaſured by the Rod, are to be reduced to a Standard Thickneſs, *viz.* of a Brick and a half thick (if it be not agreed on to the contrary); and to reduce a Wall to Standard-thickneſs, this is

The RULE.

Multiply the Number of ſuperficial Feet that are found to be contain'd in any Wall, by the Number of Half-bricks which that Wall is in Thickneſs; one third Part of that Product ſhall be the Content thereof in Feet, reduced to the Standard-thickneſs of one Brick and a half.

Example

Example 1. If a Wall be 72 Feet 6 Inches long, and 19 Feet 3 Inches high, and 5 Bricks and a half thick; How many Rods of Brick-work are contained therein when reduced to the Standard?

```
              19.25 Height.
               72.4 Length.
              ------
               9625
              3850
            13475
            ---------
            1395.625
                  11
        ----------------
          3)1535.875
          -----------
272.25) 5117.291(18 Rods.
        ----------
         239479
         -------
   68.06)21679(3 Quarters of a Rod.
         -------
          12.61
```

Anſwer, 18 Rods 3 Quarters 12 Feet.

```
      F.   I.
      72   6
      19   3
     ------
     648
     72
      18   1  6
       9   6  0
     ---------
    1395   7  6
      11
    ----
  3)15345
  -------
272)5115(18 Rods.
    ----
    2395
    ----
  68)219(3 Quarters of a Rod.
    ---
     15
```

Note, That 68.06 is one fourth Part of 272.25.

Note alſo, That in reducing of Feet into Rods, they uſually reject the odd Parts, and divide only by 272, as is done in the ſecond Way of the laſt Example; ſo the Anſwer, by that ſecond Way, is 18 Rods 3 Quarters and 15 Feet, more by about 2½ Feet than by the firſt Way, where it is done decimally; a Thing very inſignificant.

Example

Example 2. If a Wall be 245 Feet 9 Inches long, and 16 Feet 6 Inches high, and two Bricks and a half thick; I demand how many Rods of Brick-work are contained therein, when reduced to Standard-Thickneſs?

```
      245.75
        16.5
     -------
      122875
     147450
     24575
    --------
    4054.875
           5
  ----------
  3)20270
  -------
272,6756(24 Rods.
     ----
     1316
  68)228(3 Quarters of a Rod.
     ---
      24
Anſwer, 24 Rods 3 Quarters 24 Feet.
```

```
   F.   I.
  245   9
   16   6
 ---------
 1470
 245
  122  10  6
   12   0  0
 ------------
 4054  10  6 Anſwer in Feet.
```

Before

Before I ſhew how to work the two laſt Examples by Scale and Compaſſes, I will ſhew how to find proper Diviſors to facilitate the Operation; becauſe it would be too intricate and tedious to perform by Scale and Compaſſes, according to the Rule above taught.

To find proper Diviſors.

Divide 3 (the Number of Half-bricks in 1½) by the Number of Half-bricks in the Thickneſs, the Quotient will be a Diviſor, which will give the Anſwer in Feet. But if you would have a Diviſor to bring the Anſwer in Rods at once, then multiply 272.25 by the Diviſor found for Feet, and the Product will be a Diviſor which will give the Anſwer in Rods.

Example. Let it be required to find a Diviſor proper to reduce a Wall of three Bricks thick.

Divide 3 by 6, (the Half-bricks in the Thickneſs) and the Quotient is .5, which is a Diviſor that will give the Anſwer in Feet. Then multiply 272.25 by .5, and the Product is 136.125, the Diviſor, which will give the Anſwer in Rods; that is, as 136.125 is to the Length of the Wall, ſo is the Height to the Content in Rods. Or, as .5 is to the Length, ſo is the Height to the Content in Feet.

After the ſame manner you may find Diviſors for any other Thickneſs, which you will find to be as expreſs'd in the following little Table.

The Thicknefs of the Wall.	Divifors for the Anfwer in Feet.	Divifors for bringing the Anfwer in Rods.
1 Brick thick	1.5	408.375
1 $\frac{1}{2}$ Brick thick	1.	272.25
2 Bricks thick	.75	204.1875
2 $\frac{1}{2}$ Bricks thick	.6	163.35
3 Bricks thick	.5	136.125
3 $\frac{1}{2}$ Bricks thick	.4285	116.659
4 Bricks thick	.375	102.0937

Let the fecond Example, aforegoing, be wrought by Scale and Compaffes, where the Length is 245.75, the Height 16.5, and the Thicknefs 2 $\frac{1}{2}$ Bricks.

Extend the Compaffes from 163.35, (the tabular Number againft 2 $\frac{1}{2}$ Bricks) to 245.75 ; that Extent will reach from 16.5 to 24 Rods and 8 Tenths.

Again, if the Length be 75 Feet 6 Inches, and the Height 18 Feet 9 Inches, at 3 $\frac{1}{2}$ Bricks thick ; How many Rods are contained therein?

Extend the Compaffes from 116.678 (the tabular Number) to 18.75, that Extent will reach from 75.5 to 12.13 ; that is 12 Rods and a little above half a Quarter.

It will be very proper and commodions, for fuch as have frequent Occafion to meafure Brick-work, to have in the Line of Numbers little Brafs Centre-pins at each of the Numbers in the third Column of the above little Table, with a Figure to denote the Thicknefs of the Wall.

If a Wall be 104 Feet 9 Inches long, and 17 Feet 3 Inches high, how many Rods are contain'd therein?

	F.	I.	
104.75	104	9	
17.25	17	3	
52375	728		
20950	104		
73325	26	2	3
10475	12	9	0
63)1806.9375(28	1806	11	3

126

——— Anſwer, 28 Rods 42 Feet.

546
504

42

Note, That ſuch as dig Cellars, do many times do them by the Floor, 16 Feet ſquare, and a Foot deep, being a Flooɹ of Earth; that is, 324 ſolid Feet.

3. *Of Chimnies.*

If you are to meaſure a Chimney ſtanding alone by itſelf, without any Party-wall being adjoined, then girt it abont for the Length, and the Height of the Story is the Breadth; the Thickneſs muſt be the ſame as the Jambs are of, provided that the Chimney be wrought upright from the Mantle-tree to the Cieling, not deducting any thing for the Vacancy between the Floor (or Hearth) and the Mantle-tree, becauſe of the Gatherings of the Breaſt and Wings, to make room for the Hearth in the next Story.

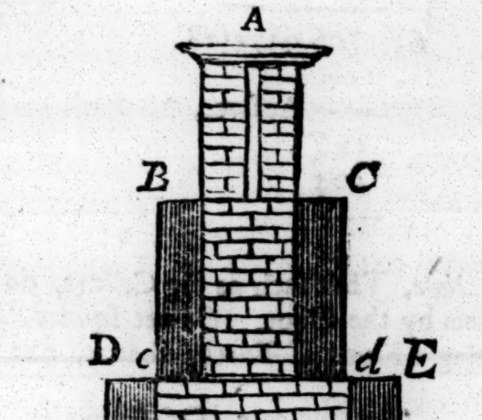
If the Chimney-back be a Party-wall, and the Wall be meaſured by itſelf, then you muſt meaſure the Depth of the two Jambs, and the Length of the Breaſt for a Length, and the Height of the Story the Breadth, at the ſame Thickneſs your Jambs were of.

When you meaſure Chimney-ſhafts, girt them with a Line round about the leaſt Place of them, for

the Length, and the Height ſhall be your Breadth: And if they be four Inch Work, then you muſt ſet down your Thickneſs at one Brick-work; but if they be wrought 9 Inches thick, (as ſometimes they are, when they ſtand high and alone above the Roof) then you muſt account your Thickneſs 1 ½ Brick, in Conſideration of Withs and Pargetting, and Trouble in Scaffolding.

It is cuſtomary, in moſt Places, to allow double Meaſure for Chimneys.

Example. Suppoſe this Figure, ABCDEFGHK, to be a Chimney that hath a double Tunnel towards the Top, and a double Shaft, and is to be meaſur'd according to double Meaſure.



Firſt, I begin with the Breaſt-wall I L, and the two Angles L K and H I, which together are 18 Feet 9 Inches; then take the Height of the Square H F, 12 Feet 6 Inches, which multiply'd together, produce 234 Feet 4 Inches 6 Parts, for the Content of the Figure FGHK.

For

For the Square DaEb, the Length of the Breast-wall and two Angles, is 14 Feet 6 Inches, and the Height Da 9 Feet; which multiply'd together, make 130 Feet 6 Inches, for the Content of the Part Da Eb.

Then the Height of the next Square 7 Feet, and the Length of the Breast-wall and two Angles is 10 Feet 3 Inches; which multiply'd together, produceth 71 Feet 9 Inches, for the Content of the Square BcCd.

The Compass of the Chimney-shafts is 13 Feet 9 Inches, and the Height 6 Feet 6 Inches; which multiply'd together, make 89 Feet 4 Inches 6 Parts, the Content of the Shafts.

The Depth of the middle Fetter, that parts the Funnels, is 12 Feet, and its Wideness 1 Foot 3 Inches; which multiply'd together, make 15 Feet, the Content thereof.

The Work.

	F.	I.			
	18	9			18.75
	12	6			12.5
	225	0			9375
	9	4	6		3750
					1875
FGHK	234	4	6	FGHK	234.375

	F.	I.		
	14	6		14.5
	9	0		9
DaEb	130	6	DaEb	130.5

	F.	I.		
	10	3		10.25
	7	0		7
BcCd	71	9	BcCd	71.75

	F.	I.	
	13	9	
	6	6	
	82	6	
	6	10	6
The Shaft	89	4	6

	13.75
	6.5
	6875
	8250
The Shaft	89.375

	F.	I.
	1	3
	12	0
The Fetter	15	0

	1.25
	12
The Fetter	15.00

	F.	I.	P.
FGHK	234	4	6
DaEb	130	6	0
BcCd	71	9	0
The Shaft	89	4	6
The Fetter	15	0	0
The Sum	541	0	0
The Double	1082	0	0

272)1082(3 Rods

68) 266(3 Quarters.

Rem. 62 Feet.

Having added the five Products together, and doubled the Sum, that double Sum is the Content of the Chimney in Feet, according to double or customary Meaſure; which Feet muſt be reduc'd to Rods, as was ſhew'd before.

So the Feet in the foregoing Example being reduc'd to Rods, (the Thickneſs being ſuppoſed 1 $\frac{1}{3}$ Bricks) it makes 3 Rods 3 Quarters and 62 Feet; that is, 4 Rods wanting 6 Feet.

This is all the Meaſure that can be allow'd, when the Chimney ſtands in a Gavel or Side-wall; in which Caſe the Back of the Chimney (here not meaſur'd) is accounted as Part of the Gavel; but if the Chimneys ſtand by themſelves, as all Stacks of Chimneys

in great Buildings do, in ſuch Caſe it is all Chimney-work, and therefore ought to be meaſured double on all Sides.

§ III. *Of* PLASTERERS *Work.*

THE Plaſterers Works are principally of two Kinds, namely, 1. Works lath'd or plaſter'd, which they call Cieling. 2. Works render'd; which is of two Kinds, *viz.* upon Brick-walls, or between Quarters, in the Partitions between Rooms; all which are meaſured by the Yard ſquare, or Square of 3 Feet, which is 9 Feet.

1. *Of Cieling.*

If a Cieling be 59 Feet 9 Inches long, and 24 Feet 6 Inches broad, how many Yards doth that Cieling contain?

Multiply 59 Feet 9 Inches by 24 Feet 6 Inches, and the Product is 1463 Feet 10 Inches 6 Parts; which divided by 9, the Quotient is 162 Yards 5 Feet.

F.	I.	
59	9	
24	6	
236		
118		
29	10	6
18	0	0
1463	10	6

```
    59.75
     24.5
    -----
    29875
   23900
  11950
 ---------
 9)1463.875
 ---------
Anſwer 162.65
```

By

By Scale and Compaſſes.

Extend the Compaſſes from 9 to 59 Feet 9 Inches, that Extent will reach from 24 Feet 6 Inches to 16.25 Yards.

2. *Of Rendering.*

Example. If the Partitions between Rooms be 141 Feet 6 Inches about, and 11 Feet 3 Inches high, How many Yards are in thoſe Partitions?

Multiply 141 Feet 6 Inches by 11 Feet 3 Inches, and the Product is 1591 Feet 10 Inches 6 Parts; which divided by 9, gives 176 Yards 7 Feet, the Anſwer.

	F.	I.	
	141	6	
	11	3	
	1556	6	
	35	4	6
	9)1591	10	6
Anſwer	176	7	

141.5
11.25
7075
2830
1415
1415
9)1591.875
176.87

Anſwer, 176.87 Yards.

Extend the Compaſſes from 9 to 141.5, that Extent will reach from 11.25 to 176.87 Yards.

Note 1. If there be any Doors, Windows, or the like, in your Partitioning, you muſt make Deductions for them.

Note 2. When you meaſure Rendering upon Brick-walls, you are to make no Deductions; but when you meaſure Rendering between Quarters, you may very well deduct one-fifth Part for the Quarters, Braces, and Entertices.

Note

Note 3. That Whiting and Colouring are both meaſured by the Yard, as Cieling and Rendering were; and as in Rendering between Quarters, you deduct one-fifth Part, ſo in Whiting and Colouring you muſt add one-fourth, or one-fifth Part at leaſt.

§ IV. *Of* JOYNERS *Work*.

JOyners do meaſure their Work by the Yard ſquare; but in taking their Dimenſions, they differ from ſome others; for they have a Cuſtom, and ſay, *We ought to meaſure where our Plane touches*: Wherefore, in taking the Height of any Room, where there is a Cornice about, and ſwelling Panels and Mouldings, they, with a String, begin at the Top, and girt over all the Mouldings; which will make the Room to meaſure much higher than it is: Then for meaſuring about the Room, they only take it as it is upon the Floor.

Example 1. If a Room of Wainſcot (being girt downwards over the Mouldings) be 15 Feet 9 Inches high, and 126 Feet 3 Inches in Compaſs, How many Yards doth that Room contain?

Multiply the Compaſs by the Height, and the Product is 1988 Feet 5 Inches 3 Parts; which divided by 9, gives 220 Yards and 8 Feet, the Anſwer.

	F.	I.	
	126	3	
	15	9	
	630		
	126		
	63	1	6
	31	6	9
	3	9	0
	9)1988	5	3
Anſwer	220	8	

126.25
15.75
63125
88375
63125
12625
9)1988.4375
220 8

Facit 220 Yards 8 Feet.

Example 2. If a Room of Wainſcot be 16 Feet 3 Inches high, (being girt over the Mouldings) and the Compaſs of the Room 137 Feet 6 Inches, How many Yards are contain'd therein?

Multiply 137 Feet 6 Inches by 16 Feet 3 Inches, and the Product is 2234 Feet 4 Inches 6 Parts; which divided by 9, the Quotient is 248 Yards and 2 Feet.

F.	I.	
137	6	
16	3	
830		
137		
34	4	6
9)2234	4	6
248	2	0

137.5
16.25
6875
2750
8250
1375
9)2234.375
248.2

Facit 248 Yards 2 Feet.

By

By Scale and Compaſſes.

For the firſt Example, extend the Compaſſes from 9 to 126.25, that Extent will reach from 15.75 to 220.9 Yards.

For the ſecond Example, extend the Compaſſes from 9 to 137.5, that Extent will reach from 16.25 to 248 Yards and about a Quarter.

In Joyners Work there is another Thing to be obſerv'd, that is, in the meaſuring of Doors, Window-ſhutters, and all ſuch Work as is wrought on both Sides, they are paid for Work and Half-work; ſo that in meaſuring all ſuch Work, you muſt firſt find the Content, as before, and take half that Content, and add to it; ſo ſhall the Sum be the Content at Work and half.

Example. If the Window-ſhutters about a Room be 69 Feet 9 Inches broad, and 6 Feet 3 Inches high, How many Yards are contain'd therein at Work and half?

Multiply 69 Feet 9 Inches by 6 Feet 3 Inches, and the Product is 435 Feet 11 Inches 3 Parts; the Half whereof is 217 Feet 11 Inches 7 Parts; which added together, the Sum is 653 Feet 10 Inches 10 Parts; which divided by 9, the Quotient is 72 Yards 5 Feet, the Content at Work and half.

F.	I.			
69	9			69.75
6	3			6.25
418	6			34875
17	5	3		13950
				41850
435	11	3		435.9375
217	11	7		217.9687
9)653	10	10		653.9062
72	5			

Facit 72 Yards 5 Feet.

By

By Scale and Compaſſes.

Extend the Compaſſes from 9 to 69.75, that Extent will reach from 6.25 to 48.4 Yards; the Half whereof is 24.02; which added together, make 72.6 Yards, the Content at Work and half.

Note, That you muſt make Deductions for all Window-lights; but you muſt meaſure the Window-boards, Sopheta-boards, and Cheeks, by themſelves.

§ V. *Of* PAINTERS *Work.*

THE taking the Dimenſions of Painters Work is the ſame as that of Joyners, by girting over the Mouldings and ſwelling Panels, in taking the Height; and it is but Reaſon that they ſhould be paid for that on which their Time and Colour are both expended. The Dimenſions thus taken, the caſting up, and reducing Feet into Yards, is altogether the ſame as the Joyners Work; but the Painter never requires Work and half, but reckons his Work once, twice, or thrice-colour'd over. Only take Notice, that Window-lights, Window-bars, Caſements, and ſuch-like Things, they do at ſo much *per* Piece.

Example. If a Room be painted, whoſe Height (being girt over the Mouldings) is 16 Feet 6 Inches, and the Compaſs of the Room 97 Feet 9 Inches, How many Yards are in that Room?

Multiply 97 Feet 9 Inches by 16 Feet 6 Inches, and the Product is 1612 Feet 0 Inches 6 Parts; which being divided by 9, the Quotient is 179 Yards and 1 Foot.

F. I.

F.	I.			
97	9			97.75
16	6			16.5
584				48875
98				58650
48	10	6		9775
9)1612	10	6		9)1612.875(179
179	1			

Facit 179 Yards 1 Foot.

By Scale and Compasses.

Extend the Compasses from 9 to 16 5, that Extent will reach from 97.75 to 179.2 Yards.

§ VI. *Of* GLASIERS *Work.*

GLasiers do measure their Work by the Foot square; so that the Length and Breadth of a Pane of Glass in Feet, being multiply'd into each other, produceth the Content.

Note, That Glasiers do usually take their Dimensions to a Quarter of an Inch; and in multiplying Feet, Inches, and Parts, the Inch is divided into 12 Parts, as the Foot is, and each Part subdivided into 12, &c.

Example 1. If a Pane of Glass be 4 Feet 8 Inches and 3 Quarters long, and 1 Foot 4 Inches 1 Quarter broad; How many Feet of Glass are in that Pane?

The Decimal of $\left\{\begin{array}{l}\text{8 Inches } \frac{3}{4} \\ \text{4 Inches } \frac{1}{4}\end{array}\right\}$ is $\left\{\begin{array}{l}.729 \\ .354\end{array}\right.$

F.	I.	P.			4.729
4	8	9			1.354
1	4	3			
					18916
4	8	9			23645
1	6	11	0		14187
	1	2	2	3	4729
6	4	10	2	3	6.403066

Anſwer, 6 Feet 4 Inches.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 1.354, that Extent will reach from 4.729 to 6.4 Feet, the Content.

Example 2. If there be 8 Panes of Glaſs, each 4 Feet 7 Inches 3 Quarters long, and 1 Foot 5 Inches 1 Quarter broad; How many Feet of Glaſs are contained in the ſaid 8 Panes?

The Decimal of $\left\{ \begin{array}{l} 7 \text{ Inches } \frac{3}{4} \\ 5 \text{ Inches } \frac{1}{4} \end{array} \right\}$ is $\left\{ \begin{array}{l} .646 \\ .437 \end{array} \right.$

F.	I.	P.			4.646
4	7	9			1.437
1	5	3			
					32522
4	7	9			13938
1	11	2	9		18584
	1	1	11	3	4646
6	8	1	8	3	6.676302
				8	8
53	5	1	6	0	53.410416

Facit 53 Feet 5 Inches.

By

By Scale and Compasses.

Extend the Compasses from 1 to 1.437, that Extent will reach from 4.646 to 6.676; then extend the Compasses from 1 to 8, that Extent will reach from 6.676 to 53.4, the Content.

Example 3. If there be 16 Panes of Glass, each 4 Feet 5 Inches and a half long, and 1 Foot 4 Inches 3 Quarters broad; How many Feet of Glass are contain'd therein?

F.	I.	P.			
					4.458
4	5	6			1.395
1	4	9			
					22290
4	5	6			40122
1	5	10	0		13374
	3	4	1	6	4458
6	2	8	1	6	6.218910
				4	4
24	10	8	6	0	24.87564
				4	4
99	6	10	0	0	99.50256

Facit 99 Feet 6 Inches.

Note, That instead of multiplying by 16, I have multiply'd by 4 twice, because 4 times 4 is 16.

By Scale and Compasses.

Extend the Compasses from 1 to 1.395, that Extent will reach from 4.458 to 6.219; then extend the Compasses from 1 to 16, that Extent will reach from 6.219 to 99.5 Feet, the Content.

Note, That when Windows have Half-rounds at the Top, they measure them at the full Height, as if they were square. Also round or oval Windows are measur'd

ſur'd at the full Length and Breadth of their Diameters. Likewiſe Crotchet-windows in Stone-work are all meaſur'd by their full Squares. And there is Reaſon for ſo doing; for the Trouble in taking their Dimenſions to work by, the Waſte of Glaſs in working, and the Time expended in ſetting up, is far more than the Glaſs can be valued at.

§ VII. *Of* MASONS *Work.*

MAſons do meaſure their Work ſometimes by the Foot ſolid, ſometimes by the Foot ſuperficial; and in ſome Places they meaſure their Walling by the Rood, that is, 21 Feet long, and 3 Feet high, which is 63 ſquare Feet. Examples of each are as follow.

Example 1. If a Wall be 97 Feet 5 Inches long, 18 Feet 3 Inches high, and 2 Feet 3 Inches thick; How many ſolid Feet are contain'd in that Wall?

F.	I.				
97	5				97.417
18	3				18.25
776					487085
97					194834
24	4	3			779336
6	0	0			97417
1	6	0			
					1777.86025
1777	10	3			2.25
2	3				
					888930125
3555	8	6			355572050
444	5	6	9		355572050
4000	2	0	9		4000.1855625

Multi-

Multiply the Length, Height, and Thickneſs together, and the laſt Product is 4000 Feet 2 Inches, the ſolid Feet contain'd in the Wall.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 18.25, that Extent will reach from 97.417 to 1777.86; then extend from 1 to 1777.86, that Extent will reach from 2.25 to 4000.18, the ſolid Content.

Example 2. If a Wall be 107 Feet 9 Inches long, and 20 Feet 6 Inches high; How many Feet ſuperficial are contain'd therein?

F.	I.		
107	9		107.75
20	6		20.5
2155	0		53875
53	10	6	215500
2208	10	6	2208.875

Facit 2208 Feet 10 Inches.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 107.75, that Extent will reach from 20.6 to 2208.875, the ſuperficial Feet.

Example 3. If a Wall be 112 Feet 3 Inches long, and 16 Feet 6 Inches high; How many Roods are contain'd therein?

F.	I.	
112	3	
16	6	
676	0	
112		
56	1	6
1852	1	6

112.25
16.5
56125
67350
11225
63)1852.125(29
592
25

Facit 29 Roods 25 Feet.

By Scale and Compaſſes.

Extend the Compaſſes from 63 to 16.5, that Extent will reach from 112.25 to 29.4 Roods, the Content.

CHAP.

CHAP. IV.

The Measuring of BOARD *and* TIMBER.

§ I. *Of* BOARD MEASURE.

TO measure a Board, is no other but to measure a long Square.

Example 1. If a Board be 16 Inches broad, and 13 Feet long; How many Feet are contain'd therein?

Multiply 16 by 13, and the Product is 208; which divided by 12, gives 17 Feet, and 4 remains, which is a third Part of a Foot.

Or thus: Multiply 156 (the Length in Inches) by 16, and the Product is 2496; which divided by 144, the Quotient is 17 Feet, and 48 remains, which is a third Part of 144, the same as before.

$$
\begin{array}{r}
12 : 13 :: 16 \\
13 \\
\hline
48 \\
16 \\
\hline
12)208 \\
\hline
17\tfrac{4}{12}
\end{array}
$$

$$
\begin{array}{l}
\text{Or, } 144 : 156 :: 16 \\
\phantom{\text{Or, } 144 : 1}16 \\
\hline
\phantom{\text{Or, } 144 : }936 \\
\phantom{\text{Or, } 144 : }156 \\
\hline
144)2496(17\tfrac{48}{144} \\
1056 \\
\hline
48
\end{array}
$$

By Scale and Compasses.

Extend the Compasses from 12 to 13, that Extent will reach from 16 to $17\frac{1}{3}$ Feet, the Content.

Or, extend from 144 to 156 (the Length in Inches) that Extent will reach from 16 to $17\frac{1}{3}$ Feet, the Content.

Example 2. If a Board be 19 Inches broad; how many Inches in Length will make a Foot?

Divide 144 by 19, and the Quotient is 7.58 very near; and so many Inches in Length, if a Board be 19 Inches broad, will make a Foot,

Inch.	Inch.	Inch.	Inch.
19 :	144 ::	1 :	7.58 *fere.*

Extend

Extend the Compasses from 19 to 144, that Extent will reach from 1 to 7.58; that is, 7 Inches and something more than a half. So, if a Board be 19 Inches broad, if you take 7 Inches, and a little more than a half with your Compasses from a Scale of Inches, and run that Extent along the Board, from End to End, you may find how many Feet that Board contains; or you may cut off from that Board any Number of Feet desir'd.

For this Purpose there is a Line upon most ordinary Joint-rules, with a little Table plac'd upon the End of all such Numbers as exceed the Length of the Rule, as in this little Table annex'd.

0	0	0	0	5	0	8½	6
12	6	4	3	2	2	1	1
1	2	3	4	5	6	7	8

Here you see, if the Breadth be one Inch, the Length must be 12 Feet; if two Inches, the Length is 6 Feet; if 5 Inches broad, the Length is 2 Feet 5 Inches, *&c.*

The rest of the Lengths are express'd in the Line, thus: If the Breadth be 9 Inches, you will find it against 16 Inches, counted from the other End of the Rule; if the Breadth be 11 Inches, than a little above 13 Inches will be the Length of a Foot, *&c.*

§ II. *Of* Squar'd Timber.

BY Squar'd Timber are here meant all such as have equal Bases, and the Sides strait and parallel. The Rules for measuring all such Solids are shew'd in § II. of Chap. 2. to which I refer you.

Example

Example 1. If a Piece of Timber be 1 Foot 3 Inches (or 15 Inches) ſquare and 18 Feet long, How many ſolid Feet are contain'd therein?

15	F.	I.	
15	1	3	
	1	3	
75			
15	1	3	
		3	9
225			
18	1	6	9
			6
1800			
225	9	4	6
			3
144)4050(28.125			
	28	1	6
1170			
180			
360			
720			

Anſwer, 28 Feet and half a Quarter.

Here, inſtead of multiplying by 18, where I wrought by Feet and Inches) I multiply'd by 6, and then by 3, becauſe 3 times 6 is 18.

Example 2. If a Piece of ſquar'd Timber be 2 Feet 9 Inches deep, and 1 Foot 7 Inches broard, and 16 Feet 9 Inches long, How many Feet of Timber are in that Piece?

Multiply the Depth, Breadth, and Length together, and the Product will be the Content.

33
19
———
297
33
———
627
16.75
———
3135
4389
3762
627
———
144)10502.25(72.93
422
1342
465
———
33

F.	I.		
2	9		
1	7		
2	9		
1	7	3	
4	4	3	
16	8	0	
69	9	0	
3	3	2	3
72	11	2	3

Anſwer 72 Feet 11 Inches; or, 72 Feet 93 Parts.

By Scale and Compaſſes.

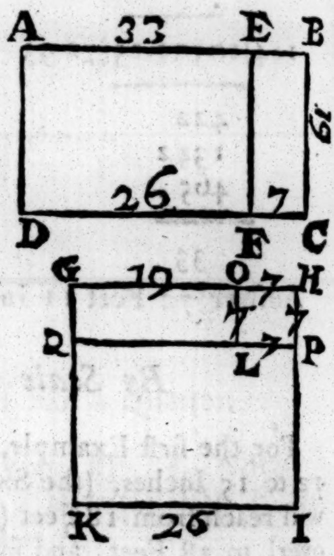
For the firſt Example, extend the Compaſſes from 12 to 15 Inches, (the Side of the Square) that Extent will reach from 18 Feet (the Length being twice turn'd over) to 28 Feet, and ſomething more.

For the ſecond Example, find a mean Proportional between 19 Inches and 33 Inches, by dividing the Space between them into two equal Parts; and the Compaſs Point will reſt upon 25, which is a mean Proportional between 19 and 33.

Then extend the Compaſſes from 12 to 25, (the Proportional found) that Extent will reach (being twice turn'd over) from 16.75 Feet, the Length, to 72.93 Feet, the Content.

A

A common Error is committed, for want of Art, in meaſuring theſe laſt Sorts of Solids, by adding the Depth and Breadth together, and taking half for the Side of a mean Square. This Error, though it be but ſmall, when the Depth and Breadth are pretty near equal; yet if the Difference be great, the Error is very conſiderable; for the Piece of Timber thus meaſur'd, will be more than the Truth, by a Piece whoſe Length is equal to the Length of the Piece of Timber to be meaſur'd, and the Square equal to half the Difference of the Breadth and Depth, as I ſhall here demonſtrate.



I ſay, the Square GHIK is greater than the Parallelogram ABCD, by the little Square OHPL; for the Parallelogram QPIK is equal to the Parallelogram AEFD; and the Parallelogram GOLQ is equal to the Parallelogram EBCF. Therefore the Square is greater than the Parallelogram by the little Square OHPL; which was to be prov'd.

Otherwiſe, you may prove it by Numbers, thus; the Sum of 33 and 19 is 52; the half thereof is 26; the Square of 26 is 676: And the Product of the Depth and Breadth is 627; the Difference of theſe two is 49, equal to the Square of half the Difference; for the Difference between 33 and 19 is 14, the Half thereof is 7, whoſe Square is 49; which was to be prov'd.

Now, if this 49 be multiply'd by the Length of the Piece, and that Product divided by 144, to bring it to Feet, and thoſe Feet added to the true Content, the Sum

Sum will be equal to the Content found by the false Way mentioned.

See the Work of both.

```
 33 Depth.          16.75 the Length.
 19 Breadth.           49 the Squ. of ½ Diff.
 ---                -----
 52 Sum.            15075
 ---                6700
 26 half            -----
 26            144)820.75(5.69
 ---                -----
156                 1007
 52                 1435
 ---                -----
676                  139
 16.75
 -----
 3380
 4732
4056
676
-------
144)11323.00(78.63
-------
1243
 910
  460
  ---
   28
```

	Feet.	
To	72.93	the true Content.
Add	5.69	the Part superfluous.
Rem.	78.62	equal to the Content by the false Way.

By Feet and Inches.

F.	I.			
0	7			
0	7			
0	4	1		
16	9			
5	5	4		
	3	0	9	
5	8	4	9	Part fuperfluous.
72	11	2	3	true Content add.
78	7	7	0	equal to the Content by the falfe Way.

	F.	I.	
	2	2	
	2	2	
	4	4	
		4	4
	4	8	4
	16	9	
	75	1	4
	3	6	3
Falfe C.	78	7	7

To find how much in Length makes a Foot of any fquar'd Timber.

Always divide 1728 (the folid Inches in a Foot) by the Area of the Bafe; the Quotient is the Length of a Foot.

This Rule is general for all Timber, which is of equal Thicknefs from End to End, whether it be fquare, triangular, multangular, or round.

Example 1. If a Piece of Timber be 18 Inches fquare, How much in Length will make a Foot folid?

```
 18
 18
---
144
18
---
324)1728(5 1/3
    1620
    ----
     108
```

Anfwer, 5 Inches and ⅓.

By

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 18, that Extent will reach from 18 to 324, the Square or Area of the Baſe; then extend from 324 to 1728, that Extent will reach down from 1 to 5 Inches, and $\frac{1}{3}$ of an Inch.

Or thus: Extend the Compaſſes from 18 to 41.569, that Extent, turn'd twice over from 1, will at laſt fall upon 5 $\frac{1}{3}$, as before.

Note, That 41.569 is the ſquare Root of 1728.

Example 2. If a Piece of Timber be 22 Inches deep, and 15 Inches broad, How much in Length will make a Foot?

```
  22
  15
  ---
 110
 22
 ---
330)1728(5.23
    ----
     780
    1200
    ----
     210
```

Anſwer, 5 Inches and .23 Parts.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 15, that Extent will reach from 22 to 330; then extend from 330 to 1728, that Extent will reach from 1 to 5.23 Inches, the Length of a Foot.

There is a Line for this Purpose upon most ordinary Rules, with a little Table at the End, of all such Numbers as exceed the Length of the Rule, such as this annex'd.

0	0	0	0	9	0	11	3	9	Inches.
144	36	16	9	5	4	2	2	1	Feet.
1	2	3	4	5	6	7	8	9	Side of the Sq.

Here you see, if the Side of the Square be 1, the Length must be 144 Feet; if two Inches be the Side of the Square, it must be 36 Feet in Length, to make a solid Foot, *&c.*

If the Side of the Square be not in the little Table, you will find it upon the Line; thus, if the Side of the Square be 16 Inches, you will find it against 6 Inches and 7 Tenths, counted from the other End of the Rule.

Then if you take the Length of a Foot from the Line of Inches with your Compasses, and run the Compasses along the Piece, from End to End, you will find how many Feet are contain'd in that Piece; or you may cut off any Number of solid Feet that shall be desir'd; but if the Sides of the Piece be unequal, find a mean proportionable Number, as is before taught, by dividing the Distance upon the Line of Numbers into two equal Parts: Thus, if the Breadth be 25 Inches, and the Depth 9 Inches, divide the Space upon the Line of Numbers, into two equal Parts, and you will find the middle Point at 15; so is 15 Inches the geometrical mean Proportional sought; then if you look for 15 upon the Line above-mention'd, you will find 7 Inches, and a little above half, to be the Length of a Foot.

§ III.

§ III. *Of unequal ſquar'd Timber.*

BY unequal ſquar'd Timber, I mean all ſuch as have unequal Baſes; that is, ſuch as is thicker at one End than at the other; and ſuch are moſt Timber-trees when they are hewn, and brought to their Squares.

The uſual Way to meaſure ſuch Timber, is to take a Square about the Middle of the Piece, which they take to be a mean Square: This Way, when the Piece is pretty near as thick at one End as at the other, is ſomething near the Truth; but when there is a great Diſproportion between the Ends of the Piece, the Error is conſiderable. All ſuch Solids being the Fruſtums of Pyramids, the true Way of meaſuring them muſt be by Sect. VII. Chap. 2. I ſhall give an Example or two, which I will work both by the true and falſe Ways, whereby you will ſee the Difference.

Example 1. If a Piece of Timber be 25 Inches ſquare at the greater End, and 9 Inches ſquare at the leſſer End, and 20 Feet long, How many Feet of Timber are in that Tree?

```
      25
       9
Sum   34
Half  17 the Side of the Square in the Middle.
      17
     119
     17
     289
      20
144)5780(40.13
    0200
     560
    128
```

Anſwer, 40.13 Feet, by the falſe Way.

By Rule II. Sect. VII. Chap. 2.

```
 25     25
  9      9
---     --
225     16 Difference of the Sides.
        16
        --
        96
       16
       ---
     3)256 the Square.
       ---
       85.333
      225
      -------
      310.333
           20
     --------------
  144)6206.660(43.101
     -------
     446
     ----
      146
        260
      -----
        116
```

Anfwer, 43.101 Feet, by the true Way; fo that there is near 3 Feet Difference.

By Scale and Compaffes.

Extend from 1 to 9, that Extent will reach from 25 (the fame Way) to 225, the Rectangle of the Sides of the two Bafes; then the Difference between the faid Sides is 16; extend from 3 to 16, that Extent will reach from 16 to 85.333, a third Part of the Square; which added to 225, the Sum is 310.333, a mean Area: then extend from 144 to 310.333, that Extent

Extent will reach from 20 (the Length) to 43.1 Foot the Content the true Way.

Extend the Compasses from 12 to 17, (the Side of the middle Square) that Extent will reach from 20, (the Length being twice turn'd over) to 40.1 Foot, the Content by the false Way.

Example 2. If a Piece of Timber be 32 Inches broad, and 20 Inches deep, at the greater End, and 10 Inches broad, and 6 deep at the lesser End, and 18 Foot long; How many Feet of Timber are in that Piece?

Rule I. Sect. VII. Chap. 2.

```
 32          6
 20         10
 ——         ——
640         60
 60
 ———
 . . .
38400      (195.959 mean Proportional
1            640   the greater Base.
———           60   the lesser Base.
29)284       ————————
 385)2300    895.959 the Sum.
 3909)37500        6 1/3 Height.
 39185)231900  ————————
 391909)3597500  5375.754
       144)5375.754(37.33
          ————
          1055
           477
           455
          ————
            23
```

And

$$\text{Add}\left\{\begin{matrix}32\\10\end{matrix}\right. \qquad \left.\begin{matrix}20\\6\end{matrix}\right\}\text{Add}$$

Sum	42	26	Sum.
Half	21	13	Half.

```
      13
      --
      63
      21
      --
     273 Area in the Middle.
      18 Length.
     ---
144)4914(34.12
    ----
    1284
    273
    ----
    594
     180
      360
    ----
      72
```

Feet.

Anſwer { Content the true Way 37.33
{ Content the falſe Way 34.12

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 20, that Extent will reach from 32 to 640, the Area of the greater Baſe.

Then extend from 1 to 10, that Extent will reach from 6 to 60, the Area of the leſſer Baſe: Then extend from 1 to 60, that Exteut will reach from 640 to 38400, the Product of the two Areas: Find the ſquare Root thereof, by dividing the Space between 1 and 38400 into two equal Parts, ſo you will find the middle Point at 195.959, the Root ſought; which is

is a mean Proportional between the greater and leſſer Areas; then add the mean Proportional and two Areas together, and the Sum is 895.959; which multiply'd by 6 (a third Part of the Length), by extending from 1 to 6, that Extent will reach from 895.959 to 5375.75. Then extend from 144 to 5375.75, and that Extent will reach from 1 to 37.33 Feet, the true Content.

For the falſe Way, half the Sum of the Breadths is 21, which is the Breadth in the Middle; and half the Sum of the Depths is 13: Extend from 1 to 13, that Extent will reach from 21 to 273, the Area of the middle Baſe: Then extend from 144 to 273, that Extent will reach from 18, the Length, to 34.12, the Content the falſe Way.

§ IV. *Of Round Timber, whoſe Baſes are equal.*

THE uſual Way to meaſure round Timber-trees is to girt them about the Middle with a String, and take the fourth Part of that Girth for the Side of a Square, by which they meaſure the Piece of Timber as if it was ſquare.

But that this is an Error, I ſhall make appear as follows. If the Circumference of a Circle be 1, the Area will be .07958; then the fourth Part of 1 is .25, which ſquar'd makes .0625; this they take for a mean Area, inſtead of .07958: Therefore the true Content always bears ſuch Proportion to the Content found by the aforeſaid cuſtomary falſe Way, as .07958 to .0625; which is nearly as 23 to 18; ſo that in meaſuring by that cuſtomary falſe Way, there is above the one fifth Part loſt, of what the true Content ought to be.

This

This Error, tho' it has been ſo often confuted, yet it is grown ſo cuſtomary in all Places, that there is little Hope of my prevailing with Men that are ſo wedded to it, to embrace the Truth: I ſhall therefore, in the following Examples, ſhew how to work both the true Way, and alſo the falſe or cuſtomary Way.

Example 1. If a Pieee of Timber be 96 Inches in Circumference or Girth, and 18 Feet long, How many Feet of Timber are contain'd therein?

A 4th Part of 96 is 24
24
———
96
48
———
576 Area Baſe,
18
———
4608
576
———
144)10368(72
1008
———
288
288
———
...

Or thus,

F.	I.
2	0
2	0
4	0
18	
72	0

Content the falſe Way, 72 Feet.

Then the true Way.

```
          96
          96
         ---
         576
        864
        ----
        9216
      .07958
      ------
       73728
      46080
     82944
    64512
    ---------
    733.40928 the Area by Prob. 5. Sect. IX. Chap. 2.
           18
    ---------
    586727424
    73340928
   -----------
144)13201.36704(94.67
   -----------
    241
     973
     1096
     ----
       88
```

The true Content 91.67 Feet.

By Scale and Compasses.

Extend from 12 to 24 (the fourth Part of the Girth), that Extent turn'd twice over from 18 Feet (the Length), will at last fall upon 72 Feet, the Content the customary Way.

Extend from 42.54 to 96 (the Girth), that Extent will reach from 18 Feet, turn'd twice over, to 91.67 Feet, the true Content.

Example

Example 2. If a Piece of Timber be 86 Inches Girth, and 20 Feet long, How many Feet are contain'd therein ?

The fourth Part of 86 is 21.5

F.	I.	P.	
1	9	6	
1	9	6	
1	9	6	
1	4	1	6
	0	10	9
3	2	6	3
			20
64	2	5	0

21.5
21.5
1075
215
430
462.25
20
144)9245.00(64.2
605
290
20

The Content the falſe Way, 64.2 Feet.

By the true Way.

86
86

516
688

7396
.07958

59168
36980
66564
51772

588.57368
20

144)11771.47360(81.74

251
1074
667

91

The true Content, 81.74 Feet.

By Scale and Compasses.

Extend from 12 to 21.5, that Extent turn'd twice over from 20, will reach at last to 64.2 Feet, the Content the false Way.

Extend from 42.54 to 86, that Extent, turn'd twice over from 20, will at last fall upon 81.74 Feet, the true Content.

These Cylindrical Proportions may be very easily wrought upon the Line of Numbers.

Problem 1. Having the Diameter of a Cylinder in Inches, to find the Length of a Foot.

Suppoſe the Diameter 22.6 Inches.

As 22.6 is to 46.9, ſo is 1 to a fourth Number; and that to the Length of a Foot in Inches 4.3.

Extend the Compaſſes from 22.6 to 46.9, that Extent will reach from 1 to a fourth Number; then turn them over again, and that will reach to 4.3 Inches.

Problem 2. Having the Diameter in Foot-meaſure, to find the Length of a Foot in Foot-meaſure.

Suppoſe the Diameter 1.88 Feet.

Then, as 1.88 is to 1.128, ſo is 1 to a fourth Number; and ſo is that to the Length of a Foot in Foot-meaſure, .358.

Extend the Compaſſes from 1.88 to 1.128, that Extent, turn'd twice from 1, will reach to .358 Parts of a Foot.

Problem 3. Having the Circumference in Inches, to find the Length of a Foot in Inches.

Suppoſe the Circumference 71 Inches.

Then as 71 is to 247.36, ſo is 1 to a fourth Number; and ſo is that to the Length of a Foot in Inches, 4.3.

Extend the Compaſſes from 71 to 147.36, that Extent, turn'd twice from 1, will reach to 4.3 Inches, the Length of a Foot.

Problem 4. Having the Circumference in Foot-meaſure, to find the Length of a Foot in Foot-meaſure.

Suppoſe the Circumference 5.92 Feet.

Then, as 5.92 is to 3.345, ſo is 1 to a fourth Number; and ſo is that to the Length of a Foot in Foot-meaſure, .358.

Extend

Extend the Compasses from 5.92 to 3.545, that Extent, turn'd twice over from 1, will fall upon .358 Parts of a Foot.

Problem 5. Having the Diameter in Inches, and the Length in Inches, to find the Content in Inches.

Suppose the Diameter 22.6 Inches, and the Length 156 Inches, or 13 Feet.

Then, as 1.128 is to 22.6; so is 156 to a fourth Number; and so is that to the Content in Inches, 62674.

Extend the Compasses from 1.128 to 22.6, that Extent, turn'd twice from 156, will fall upon 62674 Inches, the Content.

Note, That 1.128 is the Diameter when the Side of the Square is equal to 1.

Problem 6. Having the Diameter in Foot-measure, and Length in Feet, to find the Content in Feet.

Suppose the Diameter 1.88 Feet, and the Length 13 Feet.

Then, as 1.128 is to 1.88, so is 13 to a fourth Number; and so is that to the Content in Feet 36.27.

Extend from 1.128 to 1.88, that Extent, turn'd twice from 13, will fall upon 36.27.

Problem 7. Having the Diameter in Inches, and Length in Inches, to find the Content in Feet.

Suppose the Diameter 22.6 Inches, and the Length 156 Inches.

Then, as 46.9 is to 22.6, so is 156 to a fourth Number; and so is that to the Content in Feet, 36.27.

Extend from 46.9 to 22.6, that Extent, turn'd twice from 156, will fall upon 36.27 Feet, the Content.

Note, That 46.9 is the Diameter of a Circle, whose Area is 1728.

Problem 8. Having the Diameter in Inches, and Length in Feet, to find the Content in Feet.

Suppoſe the Diameter 22.6 Inches, and the Length 13 Feet.

Then, as 13.54 is to 22.6, ſo is 13 to a fourth Number: and ſo is that to the Content in Feet, 36.27.

Extend from 13.54 to 22.6, that Extent, turn'd twice from 13, will fall upon 36.27.

Note, That 13.54 is the Diameter of a Circle, when the Area is 144.

Problem 9. Having the Circumference in Inches, and Length in Inches, to find the Content in Inches.

Suppoſe the Circumference 71, and the Length 156 Inches.

Then, as 3.545 is to 71, ſo is 156 to a fourth Number; and ſo is that to 62674, the Content in Inches.

Extend the Compaſſes from 3.545 to 71, that Extent turn'd twice from 156, will fall upon 62674, the Content.

Note, That 3.545 is the Circumference, when the Side of the Square is equal to 1.

Problem 10. Having the Circumference in Feet, and Length in Feet, to find the Content in Feet.

Suppoſe the Circumference 5.92 Feet, and Length 13 Feet.

Then, as 3.545 is to 5.92, ſo is 13 to a fourth Number; and ſo is that to 36.27.

Extend from 3.545 to 5.92, that Extent, turn'd twice from 13, will fall upon 36.27 Feet, the Content.

Problem

Problem 11. Having the Circumference in Inches, and Length in Inches, to find the Content in Feet.

Suppoſe the Circumference 71 Inches, and Length 156 Inches.

Then, as 147.36 is to 71, ſo is 156, to a fourth Number; and ſo is that to the Content in Feet 36.27.

Extend the Compaſſes from 147.36 to 71, that Extent, turn'd twice from 156, will fall upon 36.27 Feet, the Content.

Note, That 147.36 is the Circumference of a Circle, whoſe Area is 1728.

Problem 12. Having the Circumference in Inches, and Length in Feet, to find the Content in Feet.

Suppoſe the Circumference 71 Inches, and Length 13 Feet.

Then, as 42.54 is to 71, ſo is 13 to a fourth Number; and ſo is that to the Content in Feet 36.27.

Extend the Compaſſes from 42.54 to 71, that Extent, turn'd twice from 13, will reach to 36.27 Feet, the Content.

Note, That 42.54 is the Circumference of a Circle, whoſe Area is 144.

§ V. *Of Round Timber, whoſe Baſes are unequal.*

THE uſual Way to meaſure round Timber, (as I ſaid before) is to take a fourth Part of the Girth in the middle of the Piece, for the Side of a mean Square. But this Way I have prov'd to be erroneous in Timber that is all the Way of an equal Thickneſs, and it muſt be much more ſo in Timber that is tapering; and the more tapering it is, the greater is the Error: for to the Error in the laſt Section, there is added the Error in the third Section; therefore, to meaſure all ſuch Timber according to Art and Truth, ſuch a Piece ought to be conſider'd as a Fruſtum of a Cone, and ſhould be meaſur'd by the Rules given in Section VIII. Chapter 2. by which Rules the following Examples are wrought.

Example 1. If a Piece of Timber be 9 Inches Diameter at the leſſer End, and 36 Inches at the other End, and 24 Feet long, How many Feet of Timber are therein?

	36	36	Subtract.
	9	9	
Rect.	324	27	Difference.
		27	
		189	
		54	
		3)729	the Square.
		243	one Third.
		324	Rectangle add.
		567	

,7854

```
              .7854
                567
              -----
              54978
             47124
            39270
            --------
A Mean Area 445.3218
                  24
            --------
            17812872
            8906436
            --------
       144)10687.7232(74.22
            -------
            607
             317
              292
             ----
                4
```

Answer, 74.22 Feet.

Or thus, by Feet and Inches.

```
F. I.         F. I.
3  0          2  3 Difference.
0  9          2  3
----          ----
2  3 Rect.    4  6
              0  6  9
              -------
              5  0  9 the Square.
              -------
              1  8  3 one Third.
              2  3  0 Rectangle added.
              -------
              3 11  3 a mean Square.
```

Then

Then, as 14 is to 11, so is 3 F. 11 I. 3 P. to the Area.

	F.	I.	P.	
	3	11	3	
			11	
7)	43	3	9	
2)	6	2	3	
	3	1	1	6
				6
	18	6	9	0
				4
	74	3	0	0

Here, instead of dividing by 14, I divide by 7 and by 2, because twice 7 is 14.

And instead of multiplying by 24 Feet, the Length, I multiply by 6 and by 4, because 6 times 4 is 24.

By Scale and Compasses this is too troublesome.

Example 2. If a Piece of Timber be 136 Inches Circumference at one End, and 32 Inches Circumference at the other End, and 21 Feet long, How many Feet of Timber are contain'd in that Piece?

136	136
32	32
272	104 Difference.
408	104
4352	416
	1040

3)10816 the Square.

3605.333 one Third.
4352 Rectangle add.

7957.333 a mean Circumf. ſquar'd.
.07958

63658664
39786665
71615997
55701331

633.24456014 the mean Area.
21

63324456014
126648912028

13298.13576294

144)13298.13(92.34

338
501
693

117
Anſwer, 92.34 Feet.

By

By Feet and Inches, thus ;

F.	I.	
11	4	
2	8	
22	8	
7	6	8
30	2	8

	F.	I.			
	8	8			Difference.
	8	8			
	69	4			
	5	9	4		
3)	75	1	4		the Square.
	25	0	5	4	
	30	2	8	0	
	55	3	1	4	the Sq. of the Circum.

	F.	I.	P.	S.	
88 : 7 ::	55	3	1	4	: the mean Area.
				7	
11)	386	9	9	4	
8)	35	1	11	9	
	4	4	8	11	the mean Area.
				7	
	30	9	2	5	
				3	
Facit	92	3	7	3	

§ VI.

§ VI. *Of the Five Regular Bodies.*

THESE Bodies may all be meaſur'd by the 4th Section of Chap. 2. except it be the Cube, or Hexaedron, which is already meaſur'd in Section I. of that Chapter.

Of the TETRAEDRON.

A Tetraedron is a Solid, contain'd under four equal and equilateral Triangles.

Let ABCD be a Tetraedron, whoſe Side is 12 Inches, the perpendicular Height 9,798 Inches.

By Sect. V. Chap. 1. the Area of the Triangle will be found 62.352; a third Part of it is 20,784, which multiply'd, by the perpendicular Height, the Product is 203.641632 ſolid Inches, the Content.

10.392 the Perpendicular of the Triangle.
6 half the Side.

62.352 Area of the Triangle.

20.784 one third Part.
9.798 the perpendicular Height.

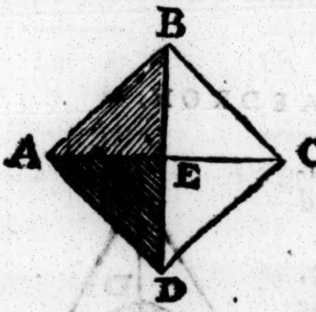
166272
187056
145488
187056

203.641632

The

The ſuperficial Content is four times the Area of the Triangle, *viz.* 249.408 Inches, becauſe there are 4 Triangles.

2. *Of the* OCTAEDRON.



The Octaedron is a Body contain'd under eight equal and equilateral Triangles.

Let ABCDE be an Octaedron, whoſe Side is 12 Inches; the Content ſolid and ſuperficial is requir'd.

An Octaedron is compos'd of two quadrangular Pyramids join'd together by their Baſes; therefore, if the Area of the Baſe be multiplied into a third Part of the Length of both Pyramids, the Product will be the ſolid Content.

5.6568	a third Part of the Length.
144	Area of the ſquare Baſe.
226272	
226272	
56568	
814.5792	the Solidity.

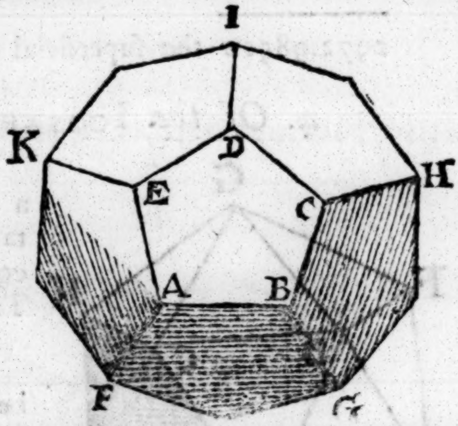
The ſuperficial Content will be juſt double to that of the Tetraedron, *viz.* 498.816, becauſe the Side of this is ſuppos'd to be equal to the Side of that, and becauſe the Octaedron is contain'd under eight Triangles, and the Tetraedron but under four.

3. *Of*

3. *Of the* DODECAEDRON.

The Dodecaedron is a solid Body contained under twelve pentangular Plains.

Let ABCDEFG HIK be a Dodecaedron, each Side thereof being 12 Inches; the Content solid and superficial is required.



The Solidity of the Dodecaedron is compos'd of 12 pentangled Pyramids, whose Vertexes all meet in the Centre. Therefore, if we find the Solidity of one of those Pyramids, and multiply that by 12, that Product will be the Solidity of the Dodecaedron.

The Altitude of one of the pentangled Pyramids will be found to be 13.36219.

The Perpendicular of the Pentagon will be

8.258292
30 half Sum of the Sides.

247.748760 Area of the Pentagon.
60454.4 A third Part of 13.36219 inverted.

99099504
9909950
1238744
99099
1486

1103.48783 Content of one Pyramid.
12

13241.85396 The Solidity of the Dodecaedron.

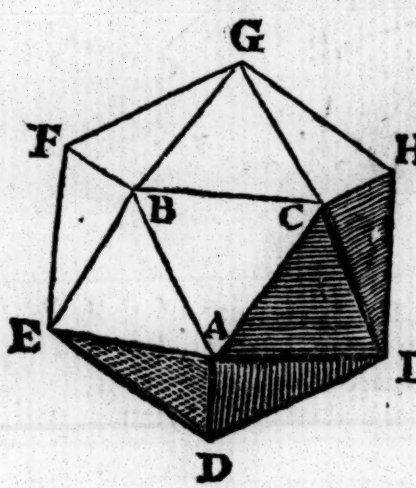
 If

If the Area of the Pentagon be multiply'd by 12. the Product will be the ſuperficial Content.

247.74876
12

2972.98512 the ſuperficial Content.

4. *Of the* Icosaedron.



The Icoſaedron is a ſolid Body, contain'd under twenty equal and equilateral Triangles.

Let ABCDEFGHI be an Icoſaedron, each Side thereof being 12 Inches; the Content ſolid and ſuperficial is required.

The Icoſaedron is compos'd of twenty triangular Pyramids, with their Vertexes all joining in the Centre.

Therefore, if the ſolid Content of one Pyramid be multiply'd by 20, the Product is the whole ſolid Content of the Icoſaedron.

10.39224 the Perpendicular of the Triangle.
6 half the Side.

62.35344
20

1247.06880

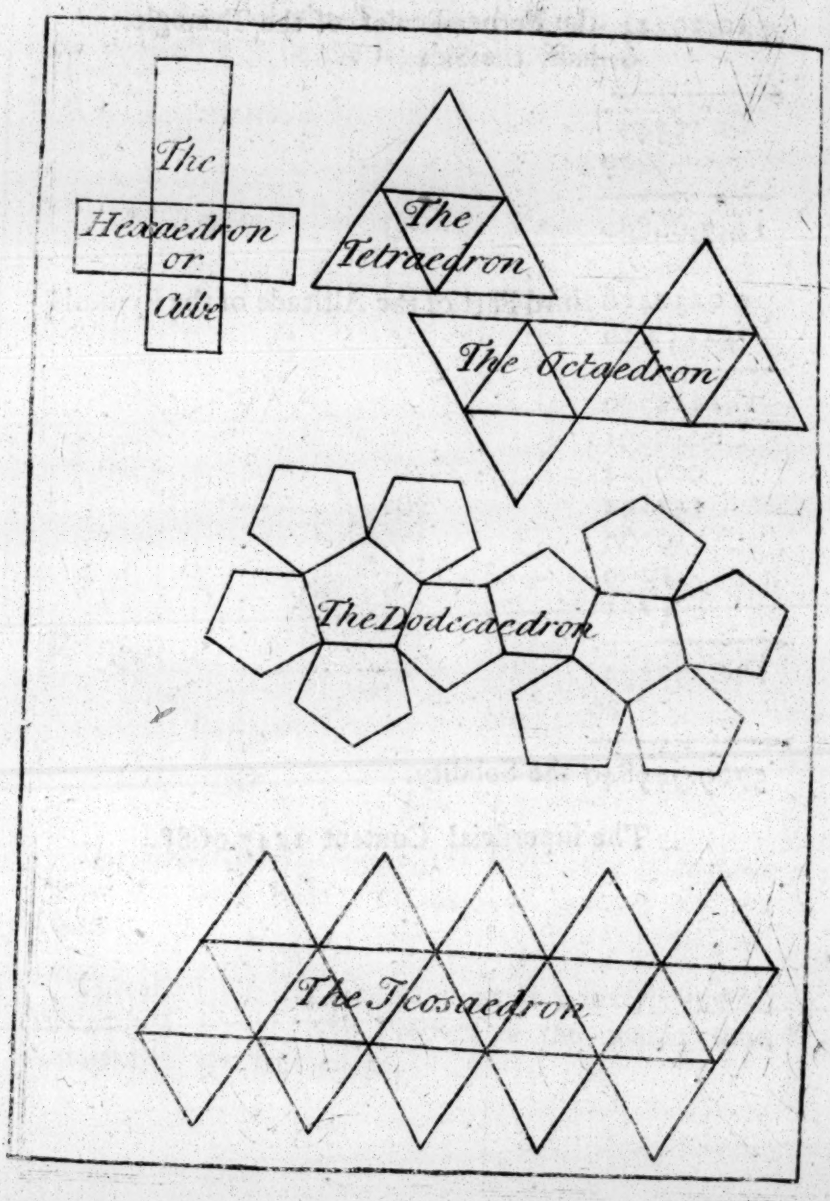
3.0230456 third Part of the Altitude of the Pyramid
44353.26

181382736
6046091
906914
151152
9069
1209
121

188.497292
20

3769.945840 the Solidity.

The superficial Content 1247.0688.



By these Figures you may cut these Bodies in fine Pastboard, cutting all the Lines half through, and so turn them up and glue them.

A TABLE *ſhewing the Solidity and Superficial Content of any of the regular Bodies, the Side being* 1, *or Unity.*

The Names of the Bodies.	The Solidity.	Superficies.
Tetraedron	0.1178511	1.732051
Octaedron	0.4714045	3.464102
Hexaedron	1.0000000	6.000000
Icoſaedron	2.181695	8.660254
Dodecaedron	7.663119	20.645729

By this Table, the Content, either ſuperficial or ſolid, of any of theſe Bodies, may very readily be found; for all like ſuperficial Figures are in Proportion one to another, as are the Squares of their like Sides: Therefore it will be, As the Square of 1 (which is 1) is to the ſuperficial Content in the Table; ſo is the Square of the Side of the like Body, to the ſuperficial Content of the ſame Body. Therefore, if the Number in the Table be multiply'd by the Square of the Side given, the Product is the ſuperficial Content requir'd.

Again, all like Solids are in ſuch Proportion to each other, as are the Cubes of their like Sides. Therefore it will be as 1 (which is the Cube of 1) is to the ſolid Content in the Table, ſo is the Cube of the Side given, to the ſolid Content requir'd. Therefore, if the Number in the Table be multiply'd by the Cube of the given Side, the Product will be the ſolid Content of the ſame Body.

Example 1. If the Side of a Dodecaedron be 12 Inches (as before), what is the Content folid and fuperficial?

7.663119	the tabular Number.
1728	the Cube of the Side.
61304952	
15326238	
53641833	
7663119	
13241.869632	the folid Content, nearly the fame as (before.

20.645729	the tabular Number.
144	the Square of the Side.
82582916	
82582916	
20645729	
2972.984976	the fuperficial Content.

By Scale and Compaffes.

Extend from 1 to 12 (the Side), that Extent being turn'd three times over from 7.63119, will at laft fall upon 13241.86, &c. the folid Content.

And if you apply the fame Extent twice from 20.645729, it will at laft fall upon 2972.98, &c. the fuperficial Content.

Example 2. If the Side of an Octaedron be 20 Inches, what is the Content ſolid and ſuperficial?

.4714045 the tabular Number.
8000 the Cube of the Side.

3771.2360000 the ſolid Content.

3.464102 the tabular Number.
400 the Square of the Side.

1385.640800 the ſuperficial Content.

By Scale and Compaſſes.

Extend from 1 to 20, that Extent, turn'd three times over from .4714045, will at laſt fall upon 3771.236, the ſolid Content. The ſame Extent turn'd twice over from 3.464, &c. will at laſt fall upon 1385.64, the ſuperficial Content.

§ VII. *How to meaſure any irregular Solid.*

IF you have any Piece of Wood or Stone that is craggy and uneven, and you deſire to find the Solidity, put the Solid into any regular Veſſel, as a Tub, a Ciſtern, or the like, and pour in as much Water as will juſt cover it; then take out the Solid, and meaſure how much the Fall of the Water is, and to find the Solidity of that Part of the Veſſel.

Example

Example. Suppose a Piece of Wood or Stone to be measur'd, and suppose a Tub 32 Inches Diameter, into which let the Stone or Wood be put, and cover'd with Water; then, when the Solid is taken out, suppose the Fall of the Water 14 Inches; square 32, and multiply the Square by .7854, the Product will be 804.2496, the Area of the Base; which multiply'd by 14, the Depth or Fall of the Water, and the Product is 11259.49, *&c.* which divided by 1728, the Quotient is 6.51 Feet; and so much is the solid Content requir'd.

CHAP.

CHAP. V.

Practical Questions in MEASURING.

Question 1. IF a Pavement be 47 Feet 9 Inches long, and 18 Feet 6 Inches broad, I demand how many Yards are contained therein ?

F.	I.	
47	9	
18	6	
376	0	
47		
23	10	6
9	0	0
4	6	0
883	4	6

F. I.
47.75
18.5
23875
38200
4775
9)883.375
98.1

Answer, 98 Yards 1 Foot.

Quest. 2. There is a Room, whose Length is 21.5 Feet, and the Breadth 17.5 Feet, is to be pav'd with Stones, each 18 Inches square: I demand how many such Stones will pave it?

```
     21.5            1.5
     17.5            1.5
     ----            ---
     1075             75
    1505             15
   215               ---
   ----              2.25 Area of one Stone.
2.25)376.25(167
   --------
    1512
     1625
   ------
        50 Answer, 167 Stones.
```

Quest. 3. There is a Room 109 Feet 9 Inches about, and 9 Feet 3 Inches high, which is all (except two Windows, each 6 Feet 6 Inches high, and 5 Feet 9 Inches broad) to be hung with Tapestry that is Ell-broad: I desire to know how many Yards will hang the said Room?

From the Content of the Room, subtract the Content of the Windows, and divide the Remainder by the square Feet in a Yard of Tapestry.

3.75
3
———
11.25

109.75 Length.
9 25 Breadth.
———
54875
21950
98775
———
1015.1875 Content of the Room.
74.75 Content of the Windows ſub.
———
11.25)940.4375(83.59
———
4043
6687
10625
———
500

Anſwer, 83.59 Yards.

5.75
6.5
———
2875
3450
———
37.375
2
———
74.750

Queſt. 4. If the Axis of a Globe be 27.5 Inches, I demand the Content, ſolid and ſuperficial.

3.1416
27.5
———
157080
219912
62
———
86.39.400 the Circumference.

86.394 the Circumference.
27.5 the Diameter.

431970
604758
172788

6)2375.8350 the superficial Content.

395.9725 a sixth Part.
27.5

19798625
27718075
7919450

10889.24375

Answer, { 6.3 Feet solid
16.49 Feet superficial.

Quest. 5. There is the Frustrum of a Globe, the Diameter of whose Base is 24 Inches, and the Altitude thereof is 10 Inches; what is the Content solid and superficial?

Find the Superficies as is directed in Page 188, and 24 the Solidity by the first Theorem in Page 190.

24	.7854	.7854	20
96	576	400	20
48			
576	47124	314.16000	400
	54978		
	39270		

452.3904 } add.
314.16 }

766.[illegible]504 the Curve Superficies.
452.3904 the Base add

1218.9408 { the whole superficial Content in Inches.

```
12 × 12 = 144
            3
         ----
          432
          100 the Square of the Alt. add.
         ----
          532 the Sum.
           10 multiply by the Alt.
         ----
         5320
         5236 multiply.
         ----
        31920
       15960
      10640
     26600
     ---------
     2785.5520 the Solidity in Inches.
```

Quest. 6. If a Tree girt 18 Feet 6 Inches, and be 24 Feet long; how many Tons of Timber are contain'd in that Tree?

```
        F.   I.
     4)18    6 the Girth.
     ---------
      4   7   6 a 4th Part
      4   7   6
     ----------
     18   6   0
      2   8   4   6
          2   3   9
     ----------------
     21   4   8   3      Here I multiply by
                  6      6 and by 4, because 6
     ----------------    times 4 is 24.
    128   4   1   6
                  4
     ----------------
  4|0)51|3  4   6   0
     ---------
     12  33
```

Answer, 12 Tons 33 Feet 4 Inches 6 Parts.

Note,

Note, That 40 Feet of Timber is a Ton, and 50 Feet a Load.

Note also, That 4 Feet broad, 4 Feet deep, and 8 Feet long, is a Chord of Fire-wood, that is, 128 cubical Feet.

Quest. 7. There is a Cellar to be dug by the Floor, whose Length is 33 Feet 7 Inches, and the Breadth is 18 Feet 9 Inches, and its Depth is to be 5 Feet 9 Inches; I demand how many Floors of Earth are in that Cellar.

	F.	I.		
	33	7		the Length.
	18	9		the Breadth.
	264			
	33			
	16	9	6	
	8	4	9	
	9	0	0	
	1	6	0	
	629	8	3	
	5	9	0	the Depth.
	3148	5	3	
	314	10	1	
	157	5	0	
324)	3620	8	4(11	
	380			
	56			

Answer, 11 Floors 56 Feet.

Note, That 18 Feet square, and a Foot deep, is a Floor of Earth, that is, 324 solid Feet.

Quest.

Quest. 8. There is a Roof covered with Tiles, whose Depth on both Sides (with the usual Allowance at the Eaves) is 35 Feet 6 Inches, and the Length 48 Feet 9 Inches; How many Squares of Tiling are contain'd therein?

	F.	I.	
	48	9	
	35	6	
	240		
	144		
	24	4	6
	17	6	0
	8	9	0
17	30	7	6

Answer, 17 Squares 30 Feet.

Quest. 9. There is a Cone, whose Diameter at the Base is 42 Inches, and the perpendicular Height 94 Inches, and it is requir'd to cut off two solid Feet from the top End thereof; I demand what Length upon the Perpendicular must be cut off?

```
      42      1728        94
      42         2        94
     ---     -----       ---
      84      3456       376
     168                846
    ----               -----
    1764               8836 Square.
   .7854                 94
   -----              ------
    7056              35344
   8820              79524
  14112              ------
 12348               830584 the Cube.
 ---------
 1385.4456
        94
 ---------
  55417824
124690104
------------
3)130231.8864
------------
  43410.6288
```

All the ſolid Bodies are in triplicate Reaſon of their homologous Sides by *Euc.* 12, 12; 12, 18; and 11, 33: thereſore it will be,

```
Solidity of the Cone.   Cub. Alt.   Solidity of 2 Feet.
   43410.6288      :     830584   : :  3456:
                           3456
                        -------
                        4983504
                       4152920
                      3322336
                     2491752
                     -----------
4341062880)2870498304(66124 the Cube of the
     ....  -----------                Length.
           265860576
            5396803
            1055740
             187528
```

```
  . .
66124(40.43
64
──────────
2124000 Resolvend.
──────────
  12
48
─────
492 Divisor.
───────
   120
4800
──────
48120 Divisor.
─────
     64
   1920
19200
──────
1939264 Subtrahend.
──────────
184736000 Resolvend.
──────────
     1212
 489648
──────────
 4897692 Divisor.
──────────
           27
      10908
1468944
──────────
147003507 Subtrahend.
──────────
 37732493
```

Answer. The Length upon the Perpendicular must be 40.43 Inches. If it had been 3 Feet, the Length had been 46.29 Inches.

If two Feet were to be cut off from the Bottom, or greatest End, then from 43410.6288 subtract 3456, and the Remainder is 39954.6288. Then say,

43410.6288 : 830584 : : 39954.6288
830584

1598185152
1598185152
3196370304
1997731440
11986388640
3196370304

13410.6288)33185675407.2192(	764459(91.4
	729
279823524	35459
19359751	27
1995500	243
259075	2457
42022	271
2952	243
	24571
	10888000
	273
	24843
	248703

Answer. It must be cut at 91.4 Inches from the Top, or 2.6 Inches from the Bottom.

Quest.

Quest. 10. If a ſquare Piece of Timber be 12 Feet long, and if the Side of the Square of the greater Baſe be 21 Inches, and the Side of the Square of the leſſer Baſe be 3 Inches; How far muſt I meaſure from the greater End, to cut off 5 ſolid Feet?

Firſt, find the Length of the whole Pyramid, thus; the Difference between 21 and 3 is 18: Then,

Diff. Length. great. Length
As 18 : 12 : : 21 : 14

So I find the whole Length of the Pyramid 14 Feet, or 168 Inches.

The ſolid Content of the whole Pyramid is 24696 Inches, and the ſolid Content of 5 Feet is 8640; which ſubtracted from 24696, there remains 16056 Inches. Then, the Cube of 168 (the Length) is 4741632. Then,

24696 : 4741632 : : 16056 :
3082752, whoſe Cube Root is 145.54; ſubtract this Root from 168 (the Length), and there remain 22.46 Inches, which is the Length of 5 ſolid Feet at the great End.

Quest. 11. Three Men bought a Grinding-ſtone of 40 Inches Diameter, which coſt 20 Shillings; of which Sum the firſt Man paid 9 Shillings, the ſecond 6 Shillings, and the third 5 Shillings: I demand how much of the Stone each Man muſt grind down, proportionable to the Money he paid?

All Circles are in duplicate Reaſon of their Diameter, by *Euc.* 12, 2.

Square the Semidiameter, which makes 400. Then

s. Square. s.
20 : 400 : : 9 : 180

This 180 is the Square of the Semidiameter of the Circle belonging to the firſt Man.

s. s.
And, 20 : 400 : : 6 : 120

This 120 is the Square of the Semidiameter of the Circle belonging to the ſecond.

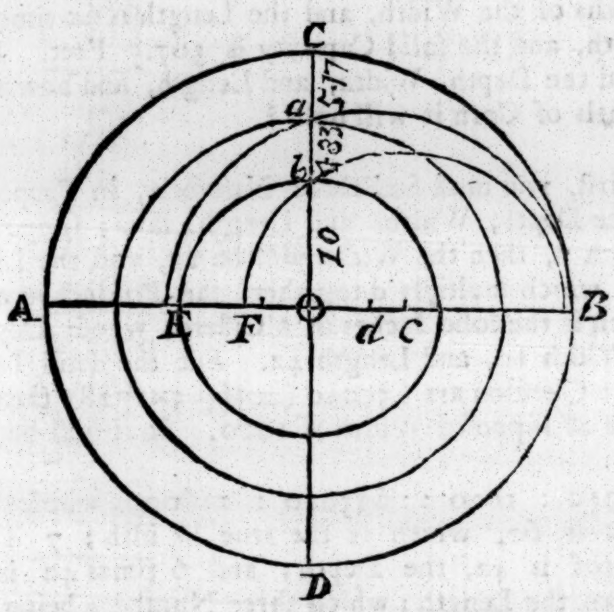
s. s.
And, 20 : 400 : : 5 : 100

This 100 is the Square of the Semidiameter of the Circle belonging to the third.

Then, from 400 (the Square of the Semidiameter of the Stone) ſubtract 180; and there remains 220, whoſe ſquare Root is 14.83 Inches; which ſubtracted from 20 Inches (the Semidiameter), there remain 5.17 Inches, which is the Breadth of the Ring, or Part of the Stone which muſt be ground down by the firſt.

Then, from 220 ſubtract 120, and there remains 100, whoſe ſquare Root is 10; ſubtract that from 14.83, and there remain 4.83 Inches, the Breadth of the Ring, or Part to be ground down by the ſecond Man. The third muſt grind down the Remainder, which is 10 Inches, the ſquare Root of 100.

This Queſtion may very eaſily and ſpeedily be perform'd Geometrically, as in the annexed Scheme.



First, upon the Centre ⊙ ſtrike the Circle ABCD, and croſs it at right Angles with the two Diameters AB and CD: Then divide the Semidiameter A ⊙, (which ſuppoſe 20) in Proportion to 9 s. 6 s. and 5 s. (the ſeveral Sums paid by the three Men) by the Points E and F; ſo ſhall AE be 9, EF 6, and F ⊙ 5: Then divide EB into 2 equal Parts in d, and upon d, as a Centre, ſtrike the Semicircle EaB; and divide FB into 2 equal Parts in c, and upon c, as a Centre, with the Radius cF, ſtrike the Semicircle FbB: So have you the Semidiameter ⊙ C divided into three ſuch Parts as the Stone ought to be divided; and Circles, ſtruck thro' thoſe Points, will ſhew how much each Man muſt grind for his Share.

Queſt. 12. A Gard'ner he had an upright Cone,
Out of which ſhould be cut him a Rolling-ſtone,
The biggeſt that e'er it could make:
The Maſon he ſaid, That there was a Rule
For ſuch Sort of Work, but he had a thick Skull:
Now help him for Pity's ſake.

Anſwer, It muſt be cut at one third Part of the Altitude.

Queſt.

Quest. 13. There is a Ciſtern, whoſe Depth is ſeven Tenths of the Width, and the Length is ſix times the Depth, and the ſolid Capacity is 367.5 Feet. I demand the Depth, Width, and Length, and how many Buſhels of Corn it will hold ?

Firſt, you muſt find three Numbers, in Proportion to the Depth, Width, and Length, thus; ſuppoſe the Depth 7, then the Width will be 10, and the Length 42; which multiply'd together, the Product is 2940, which is the ſolid Inches in a Ciſtern, whoſe Depth is 7, Width 10, and Length 42. But the ſolid Inches in the Queſtion are 635040 ($=367.5\times1728$) then the Cube of ſuppoſed Width is 1000. So it will be,

2940 : 1000 : : 635040 : 216000, whoſe Cube Root is 60, which is the true Width; 7 Tenths thereof is 42, the Depth; and 6 times 42 is 252 Inches, the Length; which three Numbers being multiply'd together, the Product will be 635040. If theſe ſolid Inches be divided by 2150.42, and the Quotient is 295 $\frac{66610}{215042}$ Buſhels, or 36 Quarters 7 Buſhels 1 Peck 4 Pints. And ſo much will the Ciſtern hold.

Quest. 14. Suppoſe, Sir, a Buſhel be exactly round,
Whoſe Depth being meaſur'd, 8 Inches is found,
If the Breadth 18 Inches and half you diſcover,
This Buſhel is legal all England over.
But a Workman would make one of another Frame;
Sev'n Inch and a half muſt be the Depth of the ſame:
Now, Sir, of what Length muſt the Diameter be,
That it may with the former in Meaſure agree ?

```
                18.5
                18.5
              ------
                 925
                1480
                185
              ------
              342.25 the Square.
               .7854
              ------
              136900
             171125
            273800
           239575
          -----------
          268.803150
          -----------
                   8
          -----------
         2150.425200 the solid Inches in a Bushel.

         7.5)2150.4252(286.72336
                     . . .
                   -----
                 650
                  504
                   542
                    175
                     252
                      270
                       450
                      ----
                         0
```

.7854)286.72336	(365.0666(19.107
	1
51103	29)265
39793	261
52360	381)406
5236	381
	38207)256600

Anſwer, The Diameter muſt be 19.107 Inches, if the Depth be 7.5 Inches.

Queſt. 15. In the midſt of a Meadow well ſtored with Graſs,
I took juſt an Acre to tether my Aſs;
How long muſt the Cord be, that feeding all round,
He mayn't graze leſs nor more than his Acre of Ground?

By Problem 10. Section IX. Chap. 1. find the Diameter of a Circle containing an Acre; half that will be the Length of the Cord.

The Work.

660 Feet, the Length of an Acre.
66 Feet, the Breadth of an Acre.

3960
3960

43560 the ſquare Feet in an Acre.

As 1 : 1.2132 : : 43560

```
         43560
        ------
        763920
       63660
      38196
     50928
     ------
     . . .
     55460.5920(235.5 Diamet.
     4              ------
     ----        117.75  half.
  43)154
     129
     ----
 465)2560
     2325
    -----
4705)23559
     23525
    ------
        34
```

Anſwer, The Chord muſt be 117 Feet and 9 Inches.

Quest. 16. A Maltster has a Kiln, that is 16 Feet 6 Inches square; but he is minded to pull it down, and build a new one, that may be big enough to dry three times as much at a time as the old one will do; I demand how much square the new one must be?

```
      16.5
      16.5
     -----
       825
      990
     165
    ------
    272.25 the Area of the old one.
         3
   -------
    .  .  .
    816.75(28.57
    4
   ---
 48)416
    384
   ----
 565)3275
     2825
    -----
 5707)45000
      39949
      -----
       5051
```

Answer, The Side of the new one must be 28 Feet, and near 7 Iuches.

Quest. 16. If a round Ciſtern be 26.3 Inches Diameter, and 52.5 Inches deep; how many Inches Diameter muſt a Ciſtern be to hold twice the Quantity, the Depth being the ſame? And how many Ale-Gallons will each Ciſtern hold?

```
      26.3
      26.3
      ----
       789
     1578
     526
    ------
    691.69 the Square.
         2
    ------
     . .
    1383.38(37.19
     9
    ---
  67)483
     469
    ----
  741)1438
       741
    ------
 7429)69700
      66861
    -------
       2839
```

The Diameter of the greater is 37.19 Inches.

```
        691.69 the Square of the lesser Cistern's Dia.
        .7854
       -------
      276676
     345845
    553352
   484183
  ---------
  543.253326 Area of the Base.
         52.5
  ----------
    2716266630
   1086506652
  2716266630
  ------------
  28520.7996150 solid Content in Inches.
  -------------
  282)28520.799(101.137 Gallons.
       ------
        320
         387
         1059
          2139
        ------
           165
```

Note, That 282 solid Inches is an Ale or Beer Gallon, and 231 a Wine Gallon.

And 359.05 is the Square of the Diameter of a Circle that will hold a Gallon of Ale at an Inch deep, and 294.12 for Wine.

You

You may find the Content in Gallons, thus: Divide the Square of the Diameter by 359.05, and multiply the Quotient by the Depth.

```
359.05)1383.38      (3.853
       -------       52.5
       306230       ------
        18990       19265
         1037       7706
                   19265
                   -------
```

The Content of the greater 202.2825 Gallons.

Quest. 18. If the Diameter of a Cask at the Bung be 32 Inches, and at the Head 25 Inches, and the Length 40 Inches; How many Ale Gallons are contain'd therein?

```
 25        32                      359
 25        32                        3
----      ----                    -----
125        64                      1077
50        96
----      ----
625       1024 Square of the Bung Diameter.
          1024 the same.
           625 Square of the Head Diameter.
         -----
  1077)2673     (2.48
        ----       40
        5190    -----
         8820   99.20
        -----
          204
```

Answer, 99.2 Gallons.

Otherwise you may find a mean Diameter, and work by Scale and Compasses, thus: Subtract 25 from 32, and there remains 7, which multiply'd by .7, the Product is 4.9, which added to 25, the Sum is 29.9. Then extend the Compasses from 18.95 to 29.9, that Extent, turn'd twice from 40 (the Length) will fall upon 99.6 Gallons; something more than before.

Quest. 19. There is a Stone 20 Inches long, 15 Inches broad, and 8 Inches thick, which weighs 217 Pounds; I demand the Length, Breadth, and Thickness of another of the same Kind and Shape, which weighs 1000 Pounds.

The Cube of 20 (the Length) is 8000. Then (by *Euc.* 11.33),

217 : 8000 : : 1000 : 36870.645, whose Cube Root is 33.28 Inches, the Length of the Stone weighing 1000 Pounds. Then say,

20 : 33.28 : : 15 : 24.96
20 : 33.28 : : 8 : 13.312

$$\text{Answer,}\left\{\begin{array}{ll}\text{The Length} & 33.28\\ \text{The Breadth} & 24.96\\ \text{The Thickness} & 13.312\end{array}\right\}\text{Inches.}$$

Quest. 20. If an iron Bullet, whose Diameter is 4 Inches, weighs 9 Pounds; What will be the Weight of another Bullet (of the same Metal) whose Diameter is 9 Inches?

The Cube of 4 is 64, and the Cube of 9 is 729. Then (by *Euc.* 12.18.),

	lb.		lb.
64 :	9 : :	729 :	102.515

	lb.	*oz.*	*dr.*
Answer, It weighs	102	8	4 *fere.*

Quest.

Queſt. 21. There is a ſquare Pyramid of Marble; each Side of its Baſe is 5 Inches, and the Height thereof 15 Inches, and its Weight is 12 Pounds and a Quarter; I demand the Weight of another like ſquare Pyramid, each Side of whoſe Baſe is 30 Inches.

The Cube of 5 is 125, and the Cube of 30 is 27000. Then (by *Euc.* 12. 12.)

	lb.		lb.
125 :	12.25	: : 27000 :	2646

Anſwer, The Weight is 2646 Pounds.

Queſt. 22. There is a Ball or Globe of Marble, whoſe Diameter is 6 Inches, and its Weight 11 Pounds; What will be the Diameter of another Globe of the ſame Marble, that weighs 500 Pounds?

The Cube of 6 is 216. Then,

lb.		lb.	
11 :	216 : :	500 :	9818.1818

Whoſe Cube Root is 21.4 Inches, the Diameter ſought.

Queſt. 23. There is a Fruſtum of a Pyramid, whoſe Baſes are regular Octagons; each Side of the greater Baſe is 21 Inches, and each Side of the leſſer Baſe is 9 Inches, and its Length is 15 Feet; I demand, how many ſolid Feet are contained therein?

4.8284 the tabular Number, Page 88.
237 the Square of a mean Side.

```
     337988      21      11
    144852        9      12
   96568        ----    ----
  ---------     189    3)144
  1144.3308      48    -----
         15     ----     48
  ---------     237
   57216540
  11443308
  ---------
144)17164.9620(119.2
  -------
  276
  1324
   289
     1
```

Answer, 119.2 solid Feet.

Quest. 24. There is a Frustum of a Cone, the Diameter of the greater Base is 36 Inches, and the Diameter of the lesser Base is 20 Inches, and the Length or Height is 215 Inches; I demand the Length and solid Content of the whole Cone, and also the solid Content of the given Frustum.

First, Find the Length of the whole Cone, thus:

```
From   36
Subtr. 20
       --
```

16 : 215 :: 36 : 483.75

So the Length of the whole Cone is 483 $\frac{3}{4}$ Inches.

Then

Then find the Content of the whole Cone.

36	1017.8784
36	52.161
216	10178784
108	6107270
	101788
1296	20357
.7854	5089
5184	1728)164132.88(94 98 Feet.
6480	
10368	8612
9072	17008
	14568
1017.8784 Area Base	
	744

Thus I find the Solidity of the whole Cone 94.98 Feet.

Then find the solid Content of the Top-part that is wanting.

```
          .7854   the Area of Unity.
            400 the Square of 20.
       ----------
      3)314.1600 Area of the lesser Base.
       ----------
         104.72   a third Part.
         268.75   Altitude of the Top-part.
       ----------
          52360
        73304
      83776
     62832
    20944
  ---------------
1728)28143.5000(16.28 Feet.
     --------
     10863
       4955
       14990
       -----
        1166
```

	Feet.
Content of the Whole	94.98
Content of the Top-piece	16.28
Content of the Frustum	78.7

Quest. 25. If the Top-part of a Cone contains 26171 solid Inches, and 200 Inches its Length, and the lower Frustum thereof contains 159610 solid Inches; I demand the Length of the whole Cone, and the Diameter of each Base.

```
 200            159610 } add.
   200           26171 }
 -------        ------
 40000          185781 the Sum.
   200
 -------
 8000000
```

26171 : 8000000 : : 185781 : 5678988 1, whose Cube Root is 384.3 Inches, the Length of the whole Cone.

Then find the Diameter of the lesser Base, thus:

```
200)26171
    ------
    130.855
          3
    -------
    392.565 Area of the lesser Base.
Then by Prob. 10. Sect. IX. Chap. 1.
1 : 1.2732 : : 392.565
                1.2732
              --------
                785130
              1177695
             2747955
             785130
            392565
           -------------
            .  .  .
           499.8137580(22.35
           4
          ------
        42)99
           84
          ------
       443)1581
           1329
          ------
      4465)25237
           22325
          ------
            2912
```

Again,

	Lesser Leng.	Less. Diam.	Gr. Leng.	Gr. Diam.
Again,	200 :	22.35 : :	384.3 :	42.94.

		Inches.
Answer,	The Length of the whole Cone	384.3
	The Diameter of the greater Base	43,94
	The Diameter of the lesser Base	22.35

Quest. 26. There is a Frustum of a Cone, whose solid Content is 20 Feet, and its Length 12 Feet; and the greater Diameter bears such Proportion to the lesser as 5 to 2; I demand the Diameters.

$5 \times 5 = 25$	3)12
$2 \times 2 = 4$	——
$5 \times 2 = 10$	4)20(5 Feet.
The Sum 39	These 5 Feet are the Triple of a mean Area.

Then, 1 : 1.27324 : : 5 : 6.3662

So the Triple Square of a mean Diameter is 6.3662.

Then, 39 : 6.3662 : : 25 : 4.080897

This 4.080897 is the Square of the greater Diameter, whose Square Root is 2.020123 Feet, which is 24.24147 Inches. Then,

5 : 24.24147 : : 2 : 9.69659

So the greater Diameter is 24.24147, and the lesser Diameter is 9.69659 Inches.

Quest. 27. There is a Room of Wainscot 129 Feet 6 Inches in Circumference, and 16 Feet 9 Inches high (being girt over the Mouldings); there are two Windows, each 7 Feet 3 Inches high, and the Breadth of each, from Cheek to Cheek, 5 Feet 6 Inches; the Breadth of the Shutters of each is 4 Feet 6 Inches; the Cheek-boards and Top and Bottom-boards of each Window

Window, taken together, is 24 Feet 6 Inches, and their Breadth 1 Foot 9 Inches; the Door-case 7 Feet high, and 3 Feet 6 Inches wide; the Door 3 Feet 3 Inches wide; I demand how many Yards of Wainscot are contain'd in that Room?

F.	I.	
129	6	
16	9	
782	0	
129		
64	9	
32	4	6
2169	1	6

F.	I.		
7	3		
4	6		
29	0		
3	7	6	
32	7	6	
16	3	9	Half.
48	11	3	
		2	
97	10	6	

F.	I.		
3	3		
7	0		
22	9		
11	4	6	Half.
34	1	6	

F.	I.	
24	6	
1	9	
4	6	
18	4	6
42	10	6
		2
85	9	0

F.	I.	
7	3	
5	6	
36	3	
3	7	6
39	10	6
		2
79	9	0

F.	I.	
3	6	
7	0	
24	6	} add.
79	9	}
104	3	

The Content of the Room	2169	1	6
The Shutters, at Work and half	97	10	6
The Door, at Work and half	34	1	6
The Cheek-boards, *&c.*	85	9	0
The Sum	2386	10	6
The Window-lights and Door-case deduct	104	3	0
	9) 2282	7	6
	253	5	

Answer, 253 Yards 5 Feet.

Quest. 28. There is a Wall which contains 18225 Cube Feet, and the Height is 5 times the Breadth, and the Length 8 times the Height; What is the Length, Breadth, and Height?

Suppose the Breadth 2, then the Height must be 10, and the Length 80; which three Numbers multiply'd together, the Product will be 1600, and the Cube of 2 is 8; then say,

1600 : 8 :: 18225 : 91.125.

Then the Cube Root of 91.125 is 4.5, which is the Breadth; then 5 times 4,5 is 22.5 the Height; and 8 times 22.5 is 180, the Length.

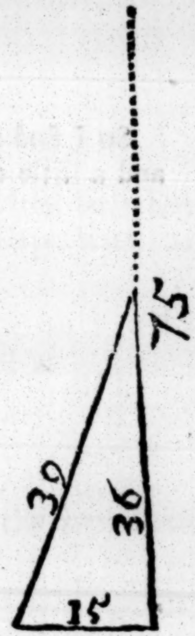
Quest. 29. There is a May-pole, whose Top end was broken off by a Blast of Wind, and the Top end, in falling, struck the Ground at 15 Feet Distance from the Foot of the May-pole, the broken Piece was 39 Feet: Now I demand the Length of the May-pole?

By *Eucl.* 1.47. the Square of the Hypothenuse of a right-angled Triangle is equal to the Sum of the Squares of the Base and Perpendicular.

There-

Therefore, from the Square of 39 ſubtract the Square of 15, the ſquare Root of the Remainder is the Piece ſtanding; to which add the Piece broken off, and you have the whole Length.

```
  39        15
  39        15
 ----      ----
 351        75
117        15
 ----      ----
1521       225
 225
 ----
1296(36
   9
 ----
66)396
   396
   ---
```



The Piece ſtanding is	36 Feet.
The Piece broken off is	39 Feet.
The whole Length	75

Queſtion 50.

A May-pole there was, whoſe Height I would know:
The Sun ſhining clear, ſtrait to work I did go:
The Length of the Shadow, upon level Ground,
Juſt ſixty-five Feet, when meaſur'd, I found;
A Staff I had there, juſt five Feet in Length;
The Length of its Shadow was four Feet one Tenth:
How high was the May-pole, I gladly would know?
And it is the Thing you're deſired to ſhow.

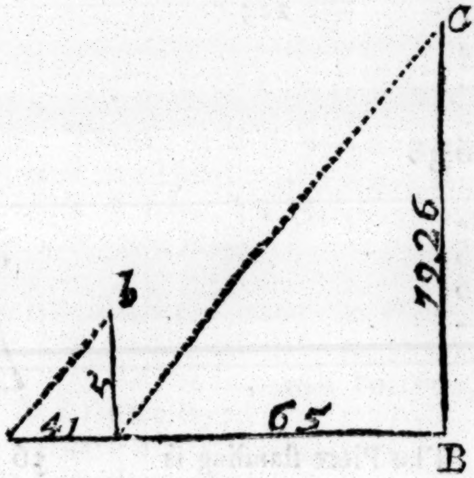
By *Eucl.* 6. 4.

a A : A b : : A B : B C.

That is,

4.1 : 5 : : 65 : 79.26.

So I find the Height of the May-pole to be 75 Feet, and a little above 3 Inches.



Here A B represents the Length of the Shadow of the May-pole, and B C the May-pole; a A the Shadow of the Staff, and A b the Staff.

Quest. 31. What will be the Diameter of a Globe, when the Solidity and superficial Content thereof are equal ?

If the Diameter be 1, the Solidity will be .5236, and the Superficies will be 3.1416; that is, as 1 to 6. And to find the superficial Content, we must multiply 3.1416 by the Square of the Axis or Diameter, and the Product is the superficial Content. And for the Solidity, multiply .5236 by the Cube of the Axis, the

the Product is the solid Content: therefore, because .5236 is a sixth Part of 3.1416, we must take 6 for the Diameter sought. For if 3.1416 be multiply'd by the Square of 6, *viz.* by 36, the Product will be 113.0976; and if .5236 be multiply'd by the Cube of 6, *viz.* by 216, the Product is likewise 113.0976, the Solidity equal to the Superficies.

Therefore, 6 is the true Answer.

Quest. 32. What will the Axis of a Globe be, when the Solidity is in Proportion to the Superficies, as 18 to 8?

Because the Solidity and Superficies is as 1 to 6; when the Axis of the Globe is 1, it will be

$$8 : 18 :: 6 : 13.5.$$

So the Diameter sought is $13\frac{1}{2}$.

If the Proportion of the Solidity to the Superficies had been as 8 to 18, then it will be

$$18 : 8 :: 6 : 2\tfrac{2}{3}.$$

So then the Diameter will be $2\frac{2}{3}$.

The Reason of these Operations, both in this and the last Question, is from Algebra.

Quest. 33. There are three Grenado-shells, of such Capacity, that the second Shell will just lie in the Concavity of the first, and the third in the Concavity of the second. The Solidity of the Metal of the first Shell is equal to its Concavity, and the Solidity of the Metal to the second, to the Concavity, is as 7 to 5; and the Solidity of the third, or least Shell's Metal, to its Concavity, is as 9 to 4. Now, supposing the Diameter of the first, or greatest Shell, to be 16

 Inches,

Inches, and allowing every ſolid Inch of Iron to weigh 4 Ounces; I demand the Diameter of the two leſſer Shells, and the Thickneſs and Solidity of Metal of every Shell, and alſo the Weight of every Shell.

The Cube of 16 is 4096; then,

1 : .5236 :: 4096 : 2144.6656.

The Half thereof is 1072.3328, which is the Solidity of the Metal of the greater Shell, as alſo the Concavity.

.5236 : 1 :: 1072.3328 : 2048.

The Cube Root of 2048 is 12.699, which is the Diameter of the ſecond Shell.

The Sum of 7 and 5 is 12; then,

12 : 5 :: 1072.3328 : 446.805.

This 446.805 is the ſolid Content of the Concavity of the ſecond.

.5236 : 1 :: 446.805 : 853.333.

The Cube Root of 853.333 is 9.485, the Diameter of the leaſt Shell.

The Sum of 9 and 4 is 13; then,

13 : 4 :: 446.805 : 137.47846.

This 137.47846 is the ſolid Content of the Concavity of the third.

.5236 : 1 :: 137.47846 : 262.5639.

The Cube Root of 262.5639 is 6.4034, the Diameter of the leaſt Shell's Concavity.

From	16	the Diameter of the greateſt,
Subtr.	12.699	the Diameter of the ſecond.
Rem.	3 301	

Half is = 1.65 the Thickneſs of Metal of the greateſt.

From

From	12.699	the Diameter of the ſecond,
Subtr.	9.485	the Diameter of the leaſt.
Rem.	3.214	

half is =1.607 the Thickneſs of Metal of the ſecond.

From	9.485	the Diameter of the leaſt,
Subtr.	6.403	the Diameter of the Concavity.
Rem.	3.082	

half is = 1.521 the Thickneſs of Metal of the leaſt.

The Metal of the greateſt is 1072.33 ſolid Inches; which divide by 4, (becauſe every ſolid Inch is a Quarter of a Pound) the Quotient is 268.08 Pounds.

The Metal of the ſecond is 625.52 ſolid Inches; which divided by 4, the Quatient is 156.38 Pounds, the Weight of the ſecond.

The Metal of the leaſt Shell is 309.32 ſolid Inches; which divided by 4, the Quotient is 77.33 Pounds, the Weight of the leaſt.

The Diam. of the	ſecond Shell	12.699	Inches.
	leaſt Shell	9.485	
The Thickneſs of the Metal of the	greateſt	1.65	Inches.
	ſecond	1.607	
	leaſt	1.541	
The Weight	greateſt	268.08	Pounds.
	ſecond	156.38	
	leaſt	77.33	

A

A SHORT APPENDIX.

§ I. *Of* Gauging.

I SHALL not here give the whole Art of Gauging (there being ſeveral Books of that Art already in Print, writ by better Hands); but ſhall only lay down ſome ſhort practical Rules, whereby any Artificer, or others, may find the Quantity of Liquor in any Veſſel upon Occaſion.

Problem I.

To find the ſeveral Multipliers, Diviſors, and Gauge-points belonging to the ſeveral Meaſures now uſed in England.

282)1.0000(.003546 Multiplier for Ale Gallons.
231)1.0000(.004329 Multiplier for Wine Gallons.
268.8)1.000(.0037202 Multiplier for Corn Gallons.
2150.42)1.000(.00046502 Multipl. for Corn Buſhels.

So, if the ſolid Inches in any Veſſel be multiply'd by the ſaid Multipliers, the Product will be Gallons in

in thr refpective Meafures; or dividing by the Divifors 282, 231, or 268.8, the Quotient will likewife be Gallons.

Note, That 282 folid Inches is a Gallon of Ale or Beer-meafure; 231 folid Inches is a Gallon of Wine-meafure; 268.8 folid Inches is a Gallon, and 2150.42 folid Inches is a Bufhel of Corn-meafure.

For circular Area's, the following Multipliers and Divifors are to be ufed.

282).785398(.002785 Multipliers for Ale Gallons.
231).785398(.003399 Multiplier for Wine Gallons.
.785398(282.(359.05 Divifor for Ale Gallons.
.785398(231.(294.12 Divifor for Wine Gallons.
.785398)2150.42(2738 Divifor for Corn Bufhels.

The fquare Root of the Divifor is the Gauge-point.

The Gauge-point for Squares in	Ale.meafure, is	16.79
	Wine-meafure, is	15.19
	Malt-bufhel, is	46.36
The Gauge-point for circular Figures in	Ale-meafure, is	18.95
	Wine-meafure, is	17.15
	Malt-bufhel, is	52.32

PROBLEM II.

To find the Area in Ale or Wine Gallons, of any rectilineal plain Figure, whether Triangular, Quadrangular, or Multangular.

TO refolve this Problem, you muft, by Chap. I. Part II. find the Area in Inches, and then bring it to Gallons, by dividing that Area in Inches by the proper Divifor, *viz.* by 282 for Ale, or by 231 for Wine; or elfe by Multiplication, by .003.546 for Ale, or by .004329 for Wine; and the Quotient or Product will be the Area.

Example

Example. Suppose a Back or Cooler in the Form of a Parallelogram, or long Square, 250 Inches in Length, and 84.5 Inches in Breadth; What is the Area in Ale or Wine Gallons?

Multiply 250 by 84.5, and the Product is 21125, the Area in Inches, which divide by 282, and the Quotient is 74.9 Gallons of Ale; or multiply'd by .003546, the Product is 74.20925 Gallons, nearly the same; and if 21125 be divided by 231, or multiply'd by .004329, it will give 91.44 Gallons of Wine.

By Scale and Compasses.

Extend the Compasses from 282 to 250, that Extent will reach from 84.5 to 74.9: And,

Extend from 231 to 250, that Extent will reach from 84.5 to 91.45.

Note, The Area's of all Superficies are always to be understood to be 1 Inch deep; otherwise it could not be said, that the Area of such a Parallelogram, Circle, &c. is so many Gallons.

Having found the Area of a Back, or Cooler, the next Thing will be to find out the true Dipping or Gauging-place in that Back, that so the true Quantity of Worts may be computed at any Depth, which may be thus done.

1. When the Bottom of the Back is cover'd all over (of any Depth) with Worts, or other Liquor, then dip it in eight or ten several Places (more or less, according to the Largeness of the Back), as remote and equally distant from each other as you can well do, noting down the wet Inches and decimal Parts of every Dip.

2. Divide the Sum of all those Dips by the Number of Places you dipp'd in, and the Quotient will be the mean Wet of all those Dips.

3. Lastly,

3. Laftly, find out fuch a Place by the Side of the Back (if you can) that juft wets the fame with that mean Dip, and make a Notch or Mark there for the true and conftant Dipping-place of that Back.

Then if any Quantity of Worts (which covers the whole Back) be deeped or gauged at that Place, and the wet Inches fo taken be multiply'd into the Area of the Back in Gallons, the Product will fhew how many Gallons of Worts are in that Back at that Time, provided the Sides of the Back do ftand at Right-angles with the Bottom.

PROBLEM III.

The Diameter of a Circle being given in Inches, to find the Area thereof in Ale or Wine Gallons.

IF the Squaro of the Diameter be multiply'd by .002785 for Ale, or by .003399 for Wine; or if it be divided by 359.05 for Ale, or by 294.12 for Wine, the Products or Quotients will he the refpective Ale or Wine Gallons.

Example. Suppofe the Diameter of a Circle be 32.6 Inches; What will be the Area in Ale or Wine Gallons?

The Square of 32.6 is 1062.76.

Then 359.05)1062.76(2.9599 Area in Ale Gallons.
And 294.12)1062.76(3.6133 Area in Wine Gall.
Or 1062.76×.002785=2.9598 Ale Gall.
And 1062.76×.003399=3 6133 Wine Gall.

By

By Scale and Compasses.

Extend the Compasses from 18.95 (the Gauge point for Ale) to 32.6 (the Diameter), that Extent will reach from 1 to a 4th Number, and from that 4th to 2.9599 Gallons. Or, extend the Compasses fr m 1 to 32.6, that Extent, turn'd twice over from .002785, will at last fall upon 2.9599.

For Wine extend from 17.15, (the Gauge-point for Wine) that Extent, turn'd twice over from 1, will at last fall upon 3.6133 Gallons.

Or thus: Extend from 1 to 32.6, that Extent will reach from .003399, being turn'd twice over, to 3.6133 Wine Gallons.

Problem IV.

The Transverse (or longest Diameter) and the Conjugate (or shortest Diameter) of an Ellipsis (or Oval) being given, to find its Area in Ale or Wine Gallons.

If the Rectangle, or Product of the two Diameters, that is, of the Length and Breadth of the Oval, be divided by 359.05, or multiply'd by .002785 for Ale, or divided by 294.12, or multiply'd by 003399 for Wine, the Quotient or Product will be the Ale or Wine Gallons requir'd.

Example. Suppose the longest Diameter be 81.4 Inches, and the shortest Diameter be 54.6 Inches, What will be the Area of that Oval?

Multiply 81.4 by 54.6, and the Product is 4444.44; then

359.05)4444.44(12.38 Area in Ale Gallons.
294.12)4444.44 15.11 Area in Wine Gallons.
Or 4444.44×.002785=12.38 Ale Gallons.
And 4444.44×.003399=15.11 Wine Gallons.

By Scale and Compasses.

First, find a mean Proportional between 81.4 and 54.6, by dividing the Distance between them into two equal Parts, and the middle Point will be at 66.6, which is the mean Proportional (that is, the Diameter of a Circle equal to the Oval). Then extend the Compasses from 18,95 (the Gauge-point for Ale) to 66.6, that Extent, turn'd twice over from 1, will at last fall upon 12.38, Ale Gallons: And extended from 17.15 (the Gauge point for Wine) to 66.6, that Extent, turn'd twice over from 1, will reach at last to 15 11 Wine Gallons.

PROBLEM V.

To find the Content in Ale or Wine Gallons of any Prism, what Form soever its Base is of.

FIRST, find its solid Content in Inches (by Sect. 1, 2, 3. of Chap. II. Part II.); then divide that Content in Inches by 282 for Ale, or by 231 for Wine; the respective Quotients will be the Content in Wine or Ale Gallons.

Otherwise, you may find the Content of a Prism by finding the Area of its Base in Gallons, (by Problem II. of this Appendix) and multiply that Area by

by the Tun's Height, or Depth within, the Product will be its Content in Gallons.

Example. Suppose a Tun, whose Base is a Parallelogram right-angled, its Length being 49.3 Inches, its Breadth 36.5 Inches, and the Depth of the Tun is 42.6 Inches; the Content in Ale and Wine Gallons is requir'd.

The Length, Breadth, and Depth, being multiply'd continually, the Product is 76656.57; which divided by 282, the Quotient is 271.83 Ale Gallons: And divided by 231, the Quotient is 331.84 Wine Gallons: And by dividing by 2150.4, such a Cistern will be found to hold 35.65 Bushels of Corn.

By Scale and Compasses.

Extend the Compasses from 282 to 36.5, the Breadth of the Base, that Extent will reach from 49.3, its Length, to 6.38 Ale Gallons, the Area of the Base; then extend from 1 to 42.6, the Depth, that Extent will reach from 6.38, the Area of the Base, to 271.8 Gallons, the Content.

Problem VI.

To find the Content of a Tun, whose Bases are alike and parallel, but unequal, being the Frustum of a Pyramid.

FIND the Area of each Base, and a mean Proportional between them, and multiply the Sum of those three by one third Part of the Depth or Height, and the Product is the Content.

Example. Suppose a Tun, whose Bases are Parallelograms; the Length of the greater is 100 Inches, and its Breadth 70 Inches; the Length of the lesser Base 80, and its Breadth 56; and the Depth of the Tun 42 Inches, the Content in Ale and Wine Gallons is requir'd.

Muliply 100 by 70, the Product is 7000, the Area of the greater Base; and 80 multiply'd by 56, the Product is 4480, the Area of the lesser Base; then multiply the two Areas into each other; and the Product is 31360000, whose square Root is 5600 a geometrical mean Proportional.

The greater Area	7000	add.
The lesser Area	4480	
The mean Proportional	5600	
	17080	
A Third of the Depth	14	
	68320	
	17080	

282)239120(847.94 A. G.
231)239120(1035.15 W. G.

PROBLEM

Problem VII.

To find the Content of a Tun, whose Bases are parallel and circular, being the Frustum of a Cone.

YOU may find the Content as in the last Problem, by multiplying the Sum of the Areas of the two Bases, and a mean Proportional, by one third Part of the Depth.

But it will be a shorter Way to find tde Area of a mean Circle in Gallons, and multiply that by the Depth, thus: To the Rectangle of the greater and lesser Diameters add one third Part of the Square of the Difference of the Diameters; that Sum is the Square of a mean Diameter, which, divided by 359.05 for Ale, or by 294.12 for Wine, gives the Area of a mean Circle in Ale or Wine Gallons, which, multiply'd by the Depth, gives the Content.

Example. Suppose the greater Diameter 80 Inches, and the lesser Diameter 71 Inches, and the Depth 34 Inches, the Con ent in Ale or Wine Gallons is requir'd.

Multiply 80 by 71, and the Product is 5680; to which add 27, (a third Part of the Square of the Difference of the Diameters) and the Sum is 5707, which is the Square of a mean Diameter; which divide by 359.05, and the Quotient will be 15.895 Gallons the Area; which multiply by 34, (the Depth) and the Product will be 540.43 Gallons the Content.

By Scale and Compasses.

Add the two Diameters together, and take half the Sum, which is 75.5, which take for a mean Diameter (though it is not exact, yet it will be near enough the Truth, if the Difference between the Diameters be not great); extend the Compasses from 18.95 (the Gauge-point for Ale) to 75.5, the mean Diameter; that Extent will reach from that 34 (the Depth) to a 4th Number, and from that to 540.4 Gallons, the Content.

And if you extend the Compasses from 17.15 (the Gauge-point for Wine) to 75.5, that Extent will reach from 34, twice turn'd over, to 659.7 Gallons of Wine.

The Method used by the Gaugers for all such Tuns is to take the Diameter in the Middle of every 10 Inches, that is, at five Inches from the Bottom, and at 15. and at 25, &c.

Then they find the Area to every one of these Diameters, and enter them in their Books. Then, when they survey, they take the wet Inches and Parts that the Liquor in the Tun is in Depth, and every ten Inches they take the respective Areas, and remove the separating Point one Place toward the Right Haud; and for what odd Inches of the Depth above the even Tens, they multiply the next Area by them, and so add all the several Products together, and the Total will be the Gallons of Liquor in the Tun.

Example. Suppose the Diameter at 5 Inches from the Bottom be 64 Inches, and at 15 Inches from the Bottom 67 Inches, and at 25 Inches 70 Inches, and at 35 Inches from the Bottom, the Diameter is 73 Inches. Now the Area answering to 64 Inches is 11.4078 Gallons; and to 67 Inches, is 12.5023 Gallons; and the Area to 70 Inches, is 13.647 Gallons; and so 73, is 14.8418 Gallons: Then, supposing .he Depth of the Liquor in the said Tun be found

found to be 3.6 Inches: Now, to cast up this Gauge; first, in the Area answering to 64 Inches, being multiply'd by 10, that is, by removing the separating Point a Place towards the Right Hand, it will be 114.078 Gallons; and the next will be 125.023; and the next 136.47 Gallons. Now these three will be the Content to 30 Inches deep. Then, to find the Content of the 3.6 Inches, multiply the next Area 14.8418 by 3.6, and the Product is 53.4305: Add all these together, and the Sum is the whole Quantity of Liquor in the Tun.

The Content at 10 Inches deep	114.078
The Content at the next 10 Inches	125.023
The Content at the next 10 Inches	136.470
The Content of the 3.6 Inches	53.430
The whole Quantity of Liquor in the Tun	429.001

PROBLEM VIII.

To find the Drip or Fall of a Tun.

SUppose the Tun last mentioned was so plac'd, that when the Bottom is but just cover'd on one Side, the Liquor is 4 Inches deep on the Side opposite: How much must be allow'd for the Fall of this Tun? That is, How much Liquor is there in the Tun?

The Diameter, in the Middle of 4 Inches from the Bottom, is 61.6 Inches; and the Area answering thereunto is 10.568; which multiply'd by 2 (that is, half 4), the Product is 21.136 Gallons; and so much Liquor will just cover the Bottom.

But

But, suppose it was set so much on one Side, as to be 30 Inches deep on one Side, when the Liquor on the opposite Side just cuts between the Bottoms and Staves; How much Liquor will there be in the Tun?

Square the Bottom Diameter, and multiply that Square by the Top Diameter, and divide the last Product by the Sum of the Diameters, and to the Quotient add the Sqnare of the Bottom Diameter, and divide the Sum by 1077.15 for Ale, or by 882.36 for Wine; multiply the Quotient by the Depth, the Product is the Content.

The Botrom Diameter of the fore mentioned Tun is 61 Inches; and the Diameter, at 30 Inches from the Bottom, is 71.5 Inches; the Square of 61 is 3721; which multiply'd by 71.5, the Product is 266051.5; this divided by 132.5, (the Sum of the Diameters) the Quotient is 2007.936; to which add 3721 (the Square of 61), and the Sum will be 5728.936; this divided by 1077.15, the Quotient is 5.3186; which multiply'd by 30 the Depth, the Product is 159.558 the Gallons of Liquor in the Tun.

When the Frustum of a Cone or Pyramid is cut, by a Diagonal Plain, through the Extremities of the Diameters, as the Liquor in the Tun represents, such Solid is called a Hoof (*Vide Ward*'s Young Mathematician's Guide, *Page* 414.)

If it be the Hoof of a square Frustum, instead of dividing by 1077.15, divide by 846 for Ale, or by 693 for Wine. All the rest of the Work is the same.

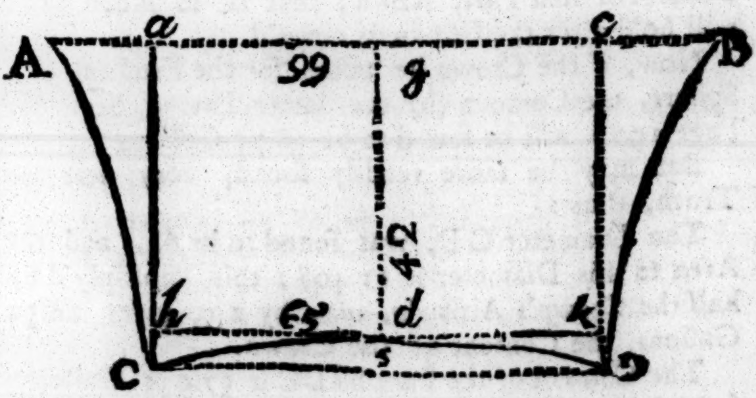
PRO-

PROBLEM IX.

To Gauge a Copper.

LET ABCD be a small Copper to be gaug'd.

Take a small Coad or Packthread, make one End fast at A, and extend the other to the opposite Side of the Copper at B, where make it fast, or cause some Person to hold it very strait; then set one End of the Instrument in the Bottom of the Copper at C, and move it to and fro, till you find the nearest Distance to the Thread (as at a): This Distance, a C, is the Depth of the Copper, which suppose to be 47 Inches.



In like manner, set the End of the Rule upon the Top of the Crown at d, and take the nearest Distance to the Thread, as d g, which suppose 42 Inches; this subtracted from a C, 47, the Remainder 5 is the Altitude of the Crown.

To find C D, the Diameter of the Bottom of the Crown.

Measure A B, the Diameter of the Top, which, admit it be 99 Inches; then hold a Thread so as a Plumbet at the End thereof may hang just over

C

C, by which means you will find the Distance A a. Do the like on the other Side; so will you find also the Distance, B; which suppose 17.5 Inches each; add these two together, and subtract their Sum (*viz.* 35) from 99, and the Remainder is 64 Inches, the Diameter at the Bottom of the Crown. The Diameter which touches the Top of the Crown, may be found by the Sliding-rule to be 65 Inches.

Now to find the Content of the Copper from the Crown upwards (that is, the Part A B k h, the Depth g d being 42 Inches, you may take the Diameter in the Middle of every 6 Inches of the Depth, which suppose to be as in the second Column of the following Table, the Numbers in the third Column are the respective Areas in Ale Gallons, found by Problem III. the Fourth Column shews the Content of every 6 Inches; all which being added together, the Sum will be the Content of that Part, ABkh; that is, so much as it will hold after the Crown is cover'd.

Now, if the Crown be taken for the Frustum of a Sphere, the Content (by the latter Part of Sect. II. Page 190.) will be found to be 28.75 Gallons.

But may be more readily found, very near the Truth, thus:

The Diameter C D, was found to be 64, and the Area to this Diameter is 11.408; this, multiply'd by half the Crown's Altitude, *viz.* by 2.5, gives 28.52 Gallons, the Content of the Crown.

The Content of the Part hkDC is 57.935 Gallons; from which subtract the Content of the Crown, 28.52, and the Remainder is 29.415 Gallons, and so much Liquor will just cover the Crown.

Par

Parts of the Depth	Diameter.	Areas.	Content of every 6 Inches.
6	95.3	25.2945	151.767
6	90.1	22.6095	135.657
6	85.0	20.1223	120.734
6	80	17.8246	106.947
6	75.2	15.7499	94.499
6	70 5	13.8426	83.056
6	66	12.1319	72.791
The Sum			765.451
To just cover the Crown			29.415
The whole Content			794.866

By Scale and Compasses.

You may find the Areas answering to every one of the Diameters, thus:

Extend the Compasses from the Gauge-point to the Diameter; that Extent, being turn'd twice over from 1, will at last fall upon the Area of that Circle: Or, being turn'd twice over from 6, will give the Content of that 6 Inches of the Depth.

Example. Extend the Compasses from 18.95 (the Gauge-point) to 95 3; that Extent, turn'd twice over from 6, will at last fall upon 151.76 Gallons, the Content of the first 6 Inches. And so of the rest.

PRO-

Problem X.

To compute the Content of any Close Cask.

IN order to perform this difficult Part of Gauging, the three following Dimensions of the Cask must be truly taken;

Viz. { The Bung-diameter, The Head-diameter, The Length of the Cask, } within the Cask.

In taking these Dimensions, it must be carefully observ'd,

1. That the Bung-hole be in the Middle of the Cask; also, that the Bung-staff, and the Staff opposite to the Bung-hole, are both regular and even within.

2. That the Heads of the Cask are equal, and truly circular; if so, the Distance between the Inside of the Chine to the Outside of its opposite Staff will be the Head-diameter within the Cask, very near.

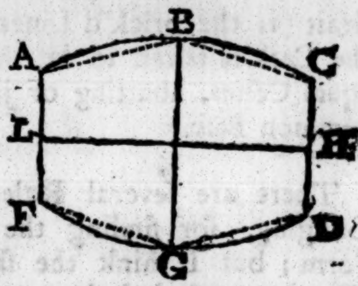
3. With a sliding Pair of Calipers, (made for that Use) take the shortest Distance, or Length, between the Outsides of the two Heads; from that Length subtract $1\frac{1}{5}$ Inch (more or less, according to the Largeness of the Cask) for the Thickness of the Head: The Remainder will be the Length of the Cask within. But if the Cask be empty, you may take the Length, by putting a strait Rod in at the Tap-hole, and allow for the Thickness of the Head.

Now by these Dimensions, one would think the Content of the Cask was perfectly limited; but it will be easy to perceive, by the following Figure,

3 that

that the Diameters and Length of one Cask may be equal to those of another, and yet one of those Casks may contain several Gallons more than the other.

As for Instance, the Figure ABCDF is suppos'd to represent a Cask: Then it is plain, that if the outward curve Lines, ABC, and FGD, are the Bounds or Staves of the Cask, it must needs hold more than if the inner prick'd Lines were the Bounds or Staves; and yet the Bung-diameter BG, and Head-diameters CD and AF, and Length LH, are the same in both those Casks.



Whence it appears, that no one general Rule can be given, whereby the Content of all Sorts of Casks can be gauged: And therefore Gaugers do usually suppose every Cask to be in some of these Forms:

1. The middle Frustum of a Spheroid.
2. The middle Frustum of a parabolic Spindle.
3. The lower Frustums of two equal parabolic Conoids.
4. The lower Frustums of two equal Cones.

1. If the Staves of the Cask be very much curved (as the outward Lines of the last Figure), then the Cask is supposed to be the middle Frustum of a Spheroid.

2. If the Staves (between the Bung and Head) be something less curved, then the Cask is taken to be the middle Frustum of a parabolic Spindle.

3. If the Staves (between the Bung and Head) be very little curved, then the Cask is taken to be the lower Frustums of two equal parabolic Conoids, abut-

ting, or joining together, upon one common Baſe.

4. If the Staves (between the Bung and Head) be ſtrait (as the prick'd Lines in the laſt Figure), then the Cask is taken to be the lower Fruſtums of two equal Cones, abutting or joining together upon one common Baſe.

There are ſeveral Rules laid down in Books of Gauging, for finding the Content of each ſeveral Form; but I think the ſhorteſt and moſt practical Way is, to find ſuch a mean Diameter, which will reduce the propoſed Cask to a Cylinder. Thus,

Multiply the Difference of the Bung and Head Diameters by .7 for the Spheroid; by .65 for the ſecond Form, by .6 for the third Form, and by .55 for the fourth Form; and add the Product to the Head-diameter, and the Sum is a mean Diameter.

Example. Suppoſe the Bung-diameter be 32 Inches, the Head-diameter 24 Inches, and the Length 40 Inches; the Content in each Variety is requir'd.

The Difference between the Bung and Head-diameter is 8; which multiply'd by .7, the Product is 5.6; which added to the Head-diameter, the Sum is 29.6, the mean Diameter: The Area anſwering thereunto will be found (by Prob. III.) to be 2.44 Ale Gallons; which multiply'd by the Length, the Product is 97.4 Gallons: and ſo much is the Content, if it be the firſt Form.

Again, if the Difference of the Diameters 8 be multiply'd by .65, the Product will be 5.2; which added to the Head-diameter, the Sum is 29.2, for the mean Diameter; and the Area anſwering thereunto is 2.3746 Gallons; which multiply'd by 40 (the Length), the Product is .94.98 Gallons, the Content, if it be of the ſecond Form.

Again, if the Difference 8 be multiplied by .6, the Product is 4.8; which added to the Head-diameter, the Sum is 28.8, the mean Diameter; the Area thereunto is 2.31 Gallons; which, multiply'd by 40, gives the Content 92.4 Gallons, for the third Form.

Again, the Difference 8, multiply'd by .55, the Product is 4.4; which added to the Head-diameter, makes the mean Diameter 28.4; the Area thereof is 2.2463; which multiply'd by 40, the Product is 89.85 Gallons, for the fourth Form.

By Scale and Compasses.

Extend the Compasses from the Gauge-point 18.95, to the first mean Diameter 29.6; that Extent will reach from the Length 40, to a fourth Number, and then to the Content, 97.4 Gallons.

Again, Extend from 28.95 to 29.2 (the second mean Diameter), that Extent, turn'd twice over from 40, will at last fall upon 94.98 Gallons.

Again, Extend from 18.95 to 28.8 (the third mean Diameter) that Extent, turn'd twice over from 40, will at last fall upon 92.4 Gallons.

Again, Extend from 18.95 to 28.4 (the fourth mean Diameter) that Extent, turn'd twice over from 40, will at last fall upon 89.85 Gallons.

Altho' I have all along made use of the Line of Numbers upon the common Two Foot, or Eighteen Inch Rules, for the Reason mentioned in the *Preface*; yet the Rules may easily be applied to the Sliding-rule, thus: To find the Area of a Circle in Gallons, set the Gauge-point upon D (that is, a single Line of Numbers) to 1 upon C (that is, a double Line); then

then againſt any Diameter upon D, is the Area upon C, thus:

To find the Content of the Caſk, laſt-mention'd, the firſt Form.

Set the Gauge-point 18.95 upon D, to the Length 40 upon C; then (againſt the mean Diameter) 29.6 upon D, is 97.4 Gallons, the Content upon C.

And againſt 29.2 (the next mean Diameter on D, is 94.98 Gallons on C.

And againſt 28.8 (the next mean Diameter) on D, is 92.4 Gallons on C.

And againſt 28.4 (the laſt mean Diameter) on D, is 89.85 Gallons on C.

All done without removing the Slider.

A TABLE

A TABLE *of the Segment of a Circle, whose Area is Unity.*

V. S.	Segm	V. S.	Se m	V. S.	Segm	V. S.	Segm
1	.0017	99	9983	26	.2056	74	.7924
2	.0048	98	.9952	27	.2178	73	.7832
3	.0087	97	.9913	28	.2292	72	.7708
4	.0134	96	.9866	29	.2407	71	.7593
5	.0187	95	.9813	30	2523	70	.7477
6	.0245	94	.9755	31	.2640	69	.7360
7	.0308	93	.9692	32	.2759	68	.7241
8	.0375	92	.9625	33	.2878	67	.7122
9	.0446	91	.9554	34	.2998	66	.7002
10	.0520	90	.9480	35	3119	65	.6881
11	.0598	89	.9402	36	3241	64	.6759
12	.0680	88	.9320	37	.3364	63	.6636
13	.0764	87	.9236	38	.3487	62	6513
14	.0851	86	.9149	39	.3611	61	6389
15	.0941	85	.9059	40	3735	60	.6265
16	.1033	84	.8967	41	.3860	59	.6140
17	.1127	83	.8873	42	.3986	58	.6014
18	.1224	82	.8776	43	.4112	57	.5888
19	.1323	81	.8677	44	.4238	56	5762
20	.1424	80	.8576	45	4364	55	5636
21	.1526	79	.8474	46	.4491	54	.5509
22	.1631	78	.1369	47	.4618	53	.5382
23	.1737	77	.8263	48	.4745	52	5255
24	.1845	76	.8155	49	.4873	51	.5127
25	.1955	75	.8045	50	.5000	50	.5000

The Use of the Table of Segments.

Is to find the Ullage, or Quantity of Liquor remaining in a Cask, whose Axis is parallel to the Horizon, the Surface of the Liquor cutting the Heads of the Cask.

The RULE *is*;

To the wet or dry Inches of the Bung-diameter, add a competent Number of Cyphers; then divide it by the whole Diameter, the Quotient found in the Table under the Title V. S. gives a Segment; which multiply'd by the whole Content of the Cask, the Product shews the Quantity of Liquor in the Cask, if the Dividend was the wet Inches, or the Ullage, if it was the dry.

Let there be a Cask in Form of a Cylinder, whose Bung-diameter is 29 Inches, the dry Part 13, and the wet 16, and the Content 80 Gallons; How many Gallons are wanting to fill the Cask?

Divide the dry Inches, 13 by 29, the Bung diameter, and the Quotient is .448; find the two first Figures .44 under V. S. and the Segment against it is .4238; to which add a proportional Part for the 8, and the whole Segment will be .4333; which multiply'd by the Content of the Cask, the Product will be 34.664 Gallons; and so much the Cask wants of being full.

Note, If the Cask be in the Form of a Cylinder, or near that Figure, the Table will give the Ullage exact enough; but if it be a spheroidal Cask, then use the following Method.

1. By

1. By the Bung and Head-diameter, find such a mean Diameter as, you judge, will reduce the propos'd Cask to a Cylinder, and then find its Content.

2. From the Bung-diameter subtract the mean Diameter, and take half the Difference.

3. From the wet Inches subtract the said Half-difference; reserve this Difference, then use this Proportion:

As the mean Diameter is to 100
(the Diameter of the tabular Circle),
so is the reserv'd Difference,
to a versed Sine in the Table.

Then, if the tabular Segment be multiply'd into the Content (as before) the Product will be the Quantity of Liquor in the Cask.

Example. Let the Cask be the same as in Page 325, of the first Form, where the Bung-diameter is 32 Inches, and the mean Diameter 29.6, and the Content 97.4 Gallons; and suppose the wet Inches 19, to find the Quantity of Liquor in the Cask.

From	32	From	19
subtr.	29.6	subtr.	1.2
Rem.	2.4	Rem.	17.8 reserv'd.
Half	1.2		

29.6 : 100 :: 17.8 : .60, the V. S.

The Segment to 60 is .6265, which multiply'd by 97.4 the Content, the Product is 61 Gallons, the Quantity of Liquor in the Cask.

If

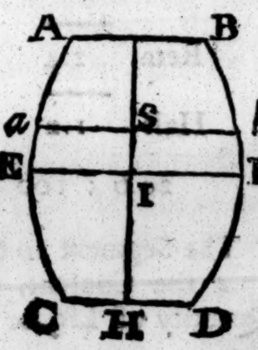
If the dry Inches had been given, by the same Method, you might have found the Ullage, or what the Cask wanted of being full.

To find what Quantity of Liquor is in a Cask, when its Axis is perpendicular to the Horizon, *viz.* when it stands upright upon one of its Heads.

To do this, you must know how to calculate the Area of any Circle, between the Bung and Head, whose Distance from the Bung, or Middle of the Cask, is given; which may be done by this Proportion.

As the Square of half the Length of the Cask is to the Difference between the Bung and Head areas; so is the Square of any Circle's Distance from the Bung, to the Difference between the Bung-area and the Area of that Circle, *viz.* the Area of the Liquor's Surface.

Then, from the Bung-area, subtract one third Part of the aforesaid Difference, *viz.* between the Bung-area and the Area of the Liquor's Surface: Multiply the Remainder by the Liquor's Distance from the Bung, and the Product will shew what Quantity of Liquor is either above or under half the Content of the Cask.



Example. Let us again suppose the Cask, in Page 325, whose Length is 40 Inches, Bung-diameter 32, and Head-diameter 24, and suppose the wet Inches, SH, 26 Inches.

The Sq. of half the Length is 400, the Distance of the Liquor's Surface from the Bung SI is 6, whose Square is 36; the Area of the Bung D 2.8519 Ale Gallons, and the Area

Area of the Head D 1.6042; the Difference 1.2477. Then,

400 : 1.2477 :: 36 : .0751

One third is = .0250

From 2.8519 Bung-area,
subtr. .0250 a Third of the Difference.

Rem. 2.8269
6 mult. Dist. from the Bung.

16.9614 Content above the Bung.
Add 48.7 Half the Content of the Cask.

65.66 The Quantity of Liquor in the G.

PROBLEM XI.

Gauging of MALT.

TO find the Quantity of Malt in a Cistern, or upon a Floor,

First, Find the Area of the Base in Bushels, by multiplying the Length by the Breadth, and dividing the Product by 2150.42, or only by 2150; and multiply that Area by the mean Depth (how to take the mean Depth, see Problem II). If the Base be circular or oval, divide by 2738 (see Problem I).

Example.

Example. There is a Ciſtern, whoſe Length is 84 Inches, and Breadth 54 Inches, and the mean Depth is 43.6 Inches; What is the Content?

Multiply 84 by 54, and the Product is 4536; which divide by 2150, and the Quotient is 2.1097 Buſhels, the Area of the Bottom at 1 Inch deep; which multiply'd by the Depth 43.6, the Product is 91.98 Buſhels, the Content.

Example. Suppoſe a Quantity of Malt upon a Floor, whoſe Length is 245 Inches, and the Breadth 184 Inches, and the mean Depth 5.6 Inches; How many Buſhels are there?

Multiply 245 by 184, and the Product is 45080; which divided by 2150, the Quotient is 20.967, the Area of the Baſe; which multiply'd by the mean Depth, the Product 117.4 Buſhels, the Content.

By the Sliding-rule.

There is an inverted Line of Numbers upon ſome Sliding-rules, mark'd with the Letter M, which was contriv'd purpoſely for Gauging of Malt; and there is a double Line of Numbers upon the Rule, and upon the Slider two double Lines of Numbers; all of theſe are of equal Radius, and all work together at once: Thus ſet the Length and Breadth againſt one another upon the inverted Line, and that which ſlides by it; then, on the other Edge of the Rule againſt the Depth, you will find the Content in Buſhels. Thus, in the firſt Example, ſet 54 upon the Slider againſt 84, upon the inverted Line; and then, againſt 43.6 upon the other Part of the Rule, is 91.98 upon the Slider.

Again, in the ſecond Example, ſet 184 upon the Slider to 245 upon the inverted Line; and againſt 5.6 upon the other Part of the Rule, is 117.4 upon the Slider.

§ II.

§ II. *Of* Land-Measuring.

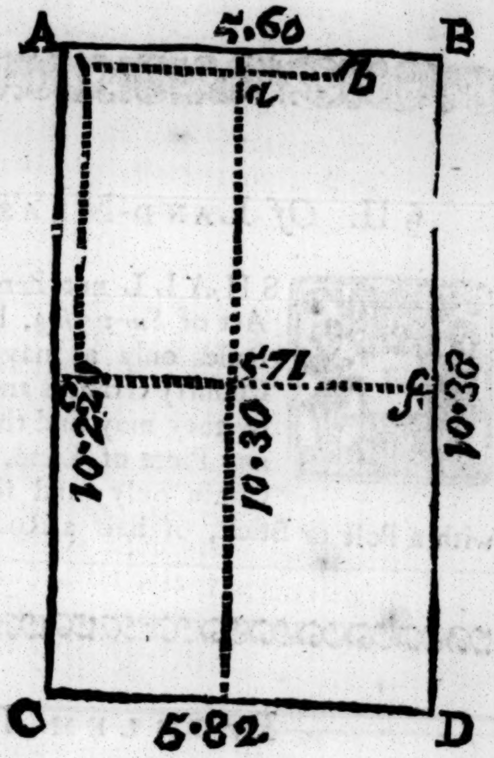
I SHALL not here give the whole Art of *Surveying*, but such practical Rules only as may be useful to the Country Grasiers and Farmers, whereby they may find the true Content of any Piece of Land, and that by the Chain only (and for want of that, with a Pole or Stick, of half a Rod in Length).

Problem I.

To find the Content of a Piece of Land in the Form of a right-angled Parallelogram, or long Square, or what is something near that Form.

TO know whether any Angle in a Field be a Right-angle, or not, you may take a Piece of Board about 4 or 5 Inches broad, and an Inch thick, either round or square; and, with a Saw, cut two Kerfs, crossing each other at Right-angles; and bore a Hole in the Middle of the Back-side, to put it upon the End of a Stick. This will represent the Instrument call'd a Cross.

Suppose



Suppose you would observe the Angle A, to know whether it be a Right-angle (or near thereunto); prick up your Stick, with the Cross upon it, a little Distance from the Fence, as at *a*; and having set up two Marks, as at *b* and *c*, of equal Distance from the Fence, turn one of the Slits directly towards *b*; and then, if the other be directly pointing to *c*, it is a Right-angle.

To measure such a Piece of Ground as this Figure above: If you measure round, and add the opposite Sides together, and take half the Sum, (if they be not equal); or else measure down about the Middle of the Length, and Middle of the Breadth; thus, the Side A B being measured, it will be 5.60 (that is, 5 Chains and 60 Links); and the opposite Side CD is 5 Chains 82 Links; the half Sum thereof is 5.71: And the Side BD is 10.38; and the Side AC 10.22; and the half Sum thereof is 10.30 (it will be the same thing, if you measure about the Middle of the Length, and Middle of the Breadth); then multiply this mean Length and mean Breadth together, *viz.* 10.30, by 5.71, and the Product is 58.8130; which divide by 10 (because 10 square Chains is an Acre) by removing the separating Point one Place towards the Left hand

Left-hand, and it will be 5.88130; that is, 5 Acres and .88130 Parts; which multiply by 4, and prick off 15 Places, and it will be 3.52520; which 3 towards the Left-hand are 3 Rods; then multiply the decimal Parts by 40, and prick off 5 Places, and it will be 21.00800; which 21 towards the Left-hand are 21 Perches.

	A.	R.	P.
So the whole Content is ———	5	3	21

See the Work.

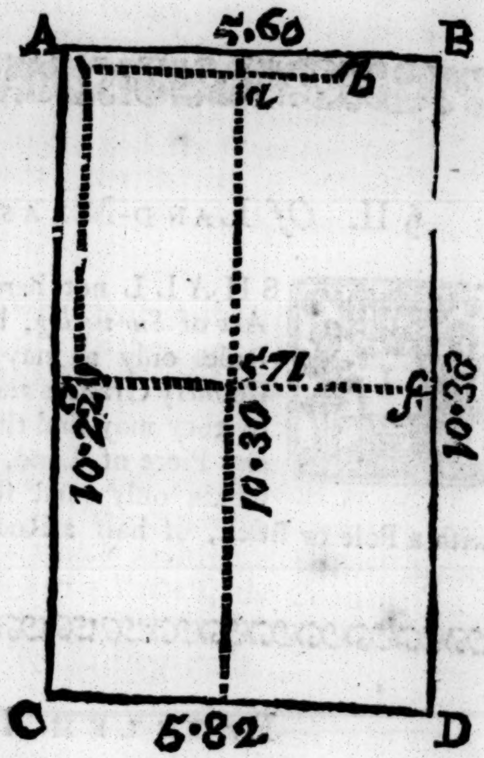
```
     5.71
    10.30
  -------
    17130
   571          A. R. P.
  -------       5  3 21
  5.88130
        4
  -------
  3.52520
       40
  -------
 21.00800
```

Note, The Chain here made use of, is 4 Poles, or Rods, in Length; the whole Chain being 100 Links.

But because every Man that may have Occasion to measure a Piece of Land, can't procure a Chain, I will therefore shew how you may measure a Piece of Land only with a Stick of half a Rod in Length; that is, 8 Feet and 3 Inches; Which Stick divide into five equal Parts, so will the whole Rod be divided into ten Parts, and will be thereby adapted to Decimal Arithmetick.

But

Suppose you would observe the Angle A, to know whether it be a Right-angle (or near thereunto); prick up your Stick, with the Cross upon it, a little Distance from the Fence, as at *a*; and having set up two Marks, as at *b* and *c*, of equal Distance from the Fence, turn one of the Slits directly towards *b*; and then, if the other be directly pointing to *c*, it is a Right-angle.



To measure such a Piece of Ground as this Figure above: If you measure round, and add the opposite Sides together, and take half the Sum, (if they be not equal); or else measure down about the Middle of the Length, and Middle of the Breadth; thus, the Side A B being measured, it will be 5.60 (that is, 5 Chains and 60 Links); and the opposite Side CD is 5 Chains 82 Links; the half Sum thereof is 5.71: And the Side BD is 10.38; and the Side AC 10.22; and the half Sum thereof is 10.30 (it will be the same thing, if you measure about the Middle of the Length, and Middle of the Breadth); then multiply this mean Length and mean Breadth together, *viz.* 10.30, by 5.71, and the Product is 58.8130; which divide by 10 (because 10 square Chains is an Acre) by removing the separating Point one Place towards the Left hand

Left-hand, and it will be 5.88130; that is, 5 Acres and .88130 Parts; which multiply by 4, and prick off 15 Places, and it will be 3.52520; which 3 towards the Left-hand are 3 Rods; then multiply the decimal Parts by 40, and prick off 5 Places, and it will be 21.00800; which 21 towards the Left-hand are 21 Perches.

	A.	R.	P.
So the whole Content is ———	5	3	21

See the Work.

```
     5.71
    10.30
  -------
    17130
   571
  -------       A. R. P.
  5.88130       5  3  21
        4
  -------
  3.52520
       40
  -------
 21.00800
  -------
```

Note, The Chain here made use of, is 4 Poles, or Rods, in Length; the whole Chain being 100 Links.

But because every Man that may have Occasion to measure a Piece of Land, can't procure a Chain, I will therefore shew how you may measure a Piece of Land only with a Stick of half a Rod in Length; that is, 8 Feet and 3 Inches; Which Stick divide into five equal Parts, so will the whole Rod be divided into ten Parts, and will be thereby adapted to Decimal Arithmetick.

But

But because each of those Parts of the Stick are something large, (each Part being 19 Inches and 8 Tenths) it will be necessary to take your Dimensions to half of one of those Parts; and then, for that half Part. set 5 in the Place of Seconds, thus; suppose 3 Parts and a half, set it down thus .35.

Problem II.

LET us suppose a Field in the Form of a long Square, whose Length is 45 Rods 5 Parts and a Half, and the Breadth 31 Rods 4 Parts and a Half: What is the Content?

Multiply the Length and Breadth together, and divide the Product by 160, (because 160 square Rods is an Acre) and the Quotient is Acres.

$$
\begin{array}{r}
45.55 \\
31.45 \\
\hline
22775 \\
18220 \\
4555 \\
13665 \\
\hline
1432.5475
\end{array}
$$

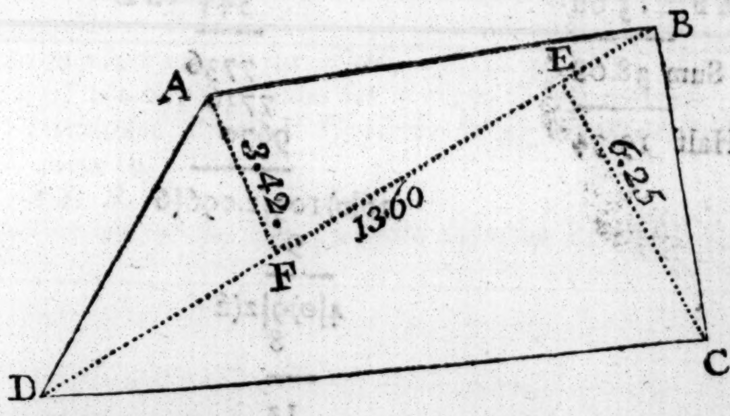
16|0)143|2(8   A. R. P.
   128   *Facit* 8 3 32

4|0) 15|2(3
   12

   32

PROBLEM III.

SUppose a Piece of Ground in the Form of a Trapezium; the Diagonal BD 13 Chains 60 Links, the Perpendicular CE 6 Chains 25 Links, and the Perpendicular AF 3 Chains 42 Links; what is the Conteut?



Multiply the Diagonal by half the Sum of the Perpendiculars. See Sect. VI. of Chap. I. Part II.

CE=6.25
AF=3.42
Sum 9.67
Half 4.83

13.60=BD
4.83
4080
10880
5440
6.56880
4
2.227520
40
11.00800

	A.	R.	P.
Facit	6	2	11

By Rods, thus ;

CE=25 Rods.
AF=13.68
Sum 38.68
Half 19.34

19.34
544=BD
7736
7736
9670
16|0)105|2.096(6
96
4|0)9|2(2
8
12

	A.	R.	P.
Facit	6	2	12

To

To take the Dimensions of the Field.

Begin at the Angle B, and measure in a direct Line towards D; but when you come at E, set up your Cross, and direct one of the Slits to D, and then look through the other Slit, and if it exactly hits the Angle C, then are you just in the Place where the Perpendicular will fall; but if it does not exactly hit the Point, move backwards, and forwards till it does so; then measure the Perpendicular, and set down the Chains and Links, or the Rods and Parts; then continue your Measure towards D; but when you come to F, set up your Cross, and try (as is above directed), whether you be in the Place where the Perpendicular will fall. Then measure the Perpendicular A F, and set down the Chain and Links or Rods and Parts; then continue your Measure to D, and set down the Measure of the whole Diagonal. This Way of Measuring is very exact and true; but the common Way used by the Grasiers and Farmers, is to measure round the Field, and to take half the Sum of the opposite Sides for a mean Side; but the last-mentioned Piece of Ground, being measur'd so, will come to

A. R. P. R. P.
7 0 22, which is 2 10 more than the Truth.

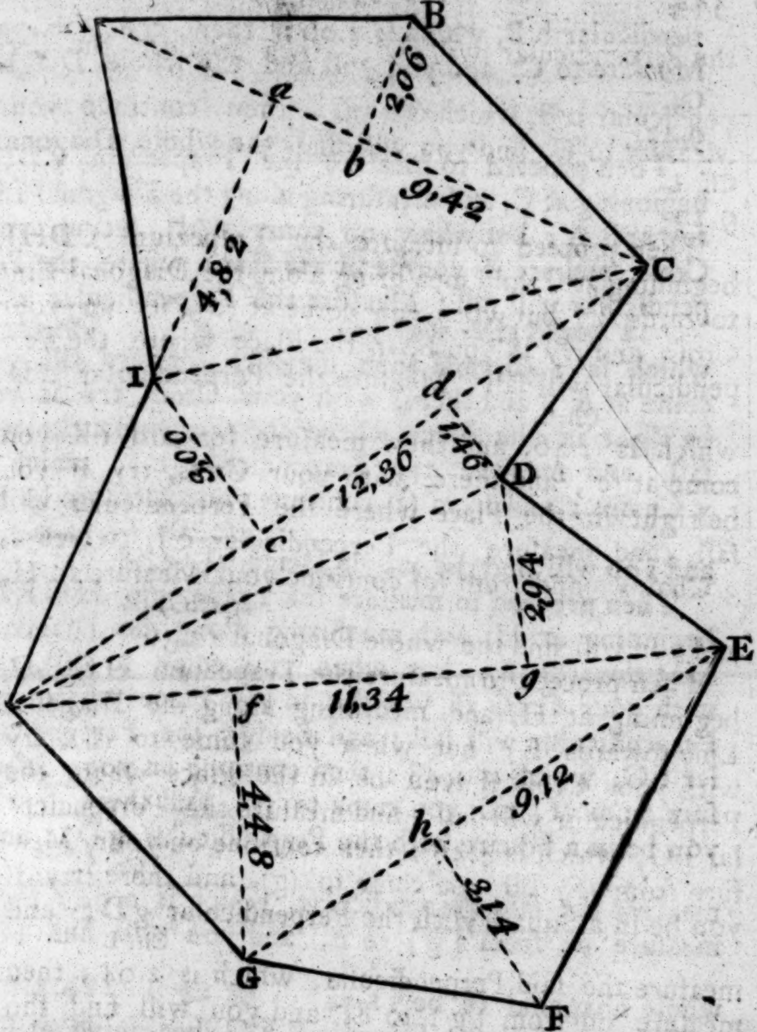
PROBLEM I.

How to measure an Irregular Field.

THE Way to measure irregular Land, is to divide it into Trapeziums and Triangles, thus:

First, view over the Field, and set up Marks at every Angle, and by those Marks you may see where to have a Trapezium, as ABCI in the following Figure.

Then



Then begin and measure in a direct Line from A towards C; but when you come to (a), set up your Cross, and try whether you be in a Square to I (as is before directed); and then measure the Perpendicular

Ch. L.

a I, which is 4.82; then measure forward again towards C, but when you come to (b) set up your Cross, and try whether you be in the Place where the

the Perpendicular will fall; then measure the Perpendicular b B, which is 2.06 Ch. L.; then continue your Measure to C, and you will find the whole Diagonal 9.42 Ch. L.

Then proceed to measure the Trapezium CDHI, beginning at C, and measuring along the Diagonal Line towards A; but when you come at (d), set up your Cross, and try if you be in the Place where the Perpendicular will fall: Measure the Perpendicular d D, which is 1.46 Ch. L., and then measure forward till you come at (c), and there, with your Cross, try if you be right in the Place where the Perpendicular will fall, and measure the Perpendicular c I, which is 3 Chains; and from (c) continue your Measure to H, and you will find the whole Diagonal 12.36 Ch. L.

Then proceed to measure the Trapezium HGED, beginning at H, and measuring along the Diagonal Line towards E; but when you come to (f), try with your Cross if you be in the Place where the Perpendiculur will fall; and measure the Perpendicular f G, which is 4.48; then continue on your Measure from (f) till you come to (g), and there try if you be in a Square with the Perpendicular g D; and measure the said Perpendicular, which is 2.94 Ch. L.; then measure on from (g) to E, and you will find the whole Diagonal to be 11.34 Ch. L.

Then measure the Triangle E F G, beginning at E, and measuring along the Base E G till you come at (h), and there with your Cross try if you be in the Place where the Perpendicular will fall; and measure the Perpendicular h F, which is 3.14 Ch. L., continue your Measure to G, and you will find the whole Base to be 9.12 Ch. L.; so you have finish'd your whole Field.

I have been the larger upon the Explanation of this Problem, because most Grounds lie in such irregular Forms.

Cast up the three Trapeziums severally, and also the Triangle; and add all the several Areas together into one Sum, which will be the Area of the whole irregular Plot.

See the Work.

See Sect. VI. Chap. I. Part II.

b B=2.06	9.42	
a I=4.82	3.44	
Sum 6.88	3768	
	3768	
half 3.44	2826	
	3.24048=Area of ABCI.	

d D=1.46	12.36	
c I =3.00	2.23	
Sum 4.46	3708	
	2472	
half 2.23	2472	
	2.75628=Area of CIHD.	

f G=4.48	11.34	
g D=2.94	3.71	
Sum 7.42	1134	
	7938	
half 3.71	3402	
	4.20714=Area of HGED.	

Base

Base=9.12	See Sect. V. Chap. I. Part II.
Half=4.56	
Perpend. 3.14	
1824	
456	
1368	

1.43184=Area of the Triangle EFG.
3.24048=Area of ABCI.
2.75628=Area of CIHD.
4.20714=Area of HGED.

Sum 11.63574=Area of the Whole.
4

2.54296
40

21.71840

	A.	R.	P.
Facit	11	2	21

F I N I S.

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